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On Estimation of Finite Population Mean Using Auxiliary Information in Presence of Measurement Errors

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Abstract: This study developed a new estimator for estimating population mean in the presence of measurement errors using auxiliary information. Data collection requires careful consideration and security precautions because it is a basic component of all statistical studies. Naturally, at the analysis step, it is assumed that all data recorded were precisely measured. There are a few circumstances, however, in which this assumption of error-free observations might not hold true. It might be possible to acquire the data with few errors. In light of this, a method for estimating the mean of a finite population in the presence of measurement errors is proposed using auxiliary information and approximate first-order equations for bias and mean squared error are also produced. It has been shown through theoretical and numerical studies that the proposed new estimator improves the one already found in the literature.
Keywords: Bias, Mean Squared Error, Efficiency and Measurement errors, Auxiliary Variable.

1. Introduction

When conducting a survey, the features of estimators based on data typically assume that the observations were gathered without measurement errors on the characteristic being researched. In reality this assumption is not always true and measurement errors like reporting and computing errors can have an impact on statistics. As a result, it is impossible to consistently estimate population parameters due to measurement inaccuracies. The parameter estimates are inaccurate and inconsistent as a result of measurement problems. The statistical inferences based on observable data continue to be valid if measurement error are very small and may be disregarded. On the other hand, if they are not insignificantly small and minor, the inferences may just be false and wrong but frequently result is not deliberate. Misra et al.(2016 a, 2016 b) , Misra et al. (2017) and Singh et al (2019) investigated a few population mean estimators under measurement errors and discussed some significant causes of measurement errors in survey data.

Let $U = U_1, U_2, \dots, U_N$ be N unit of finite population. Consider that a simple random sampling technique was used to obtain a set of n paired observations on the two variables X and Y. However, Let (x_i, yi) be the observed values rather than the true values for a simple random size n(X_i, Y_i) for the x_i, $(i = 1,2, \dots n)$represents the sampling unit in the sample as

$u_i = y_i - Y_i$ and $v_i = x_i - X_i$, where u_i and v_i are associated measurement errors that are stochastic in nature with mean zero and variances σ_u^2 and σ_v^2 respectively. Further, let u_i' and $v_i's$ are uncorrelated while $X_i's$ and $Y_i's$ are correlated. Let the population mean, of X and Y characteristics be μ_X and μ_y population variances of (X, Y) are σ_X^2 and σ_Y^2 respectively and the populations relationship between X and Y is known as correlation.

Let $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ & $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ to serve as the estimators of mean μ_X and μ_Y respectively i.e. $E(\bar{x}) = \mu_X$ and $E(\bar{y}) = \mu_Y$. However, if there are measurement errors, $s_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ and $s_Y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ are not unbiased estimators of the population variances σ_X^2 and σ_Y^2. When measurement errors are present, the expected value is given by

$E(s_y^2) = \sigma_Y^2 + \sigma_u^2$.

Given error variances σ_u^2 and σ_v^2.

$\hat{\sigma}_Y^2 = s_y^2 - \sigma_u^2 > 0$

$\hat{\sigma}_X^2 = s_x^2 - \sigma_v^2 > 0$

Further let,

$C_Y = \frac{\sigma_Y}{\mu_Y}$

$C_X = \frac{\sigma_X}{\mu_X}$

$\gamma_{2Y} = \beta_{2Y} - 3, \gamma_{2X} = \beta_{2X} - 3, \gamma_{2u} = \beta_{2u} - 3$

$\gamma_{2v} = \beta_{2v} - 3, \beta_{2Y} = \frac{\mu_4(Y)}{\mu_2^2(Y)}$

$\beta_{2X} = \frac{\mu_4(X)}{\mu_2^2(X)}, \beta_{2u} = \frac{\mu_4(u)}{\mu_2^2(u)}$

$\beta_{2v} = \frac{\mu_4(v)}{\mu_2^2(v)}, \gamma_{1(X)} = \sqrt{\beta_1(X)},\ \beta_1(X) = \frac{\mu_3^2}{\mu_2^3}$

$\mu_{qrst} = E[(X - \mu_X)^q (Y - \mu_Y)^r v^s u^t]$

$\mu_{2000} = \sigma_X^2$

$\mu_{0200} = \sigma_Y^2$

$\mu_{0020} = \sigma_v^2$

$\mu_{0002} = \sigma_u^2$

To estimate the population mean, an estimator in presence of measurement errors is proposed as

$$\hat{\bar{y}}_{ME} = \bar{y}\exp\left\{k_1\left(\frac{\hat{\sigma}_X^2}{C_X^2} - \bar{x}^2\right)\right\}\exp\left\{k_2\left(\frac{\hat{\sigma}_Y^2}{C_Y^2} - \bar{y}^2\right)\right\} \tag{1.1}$$

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2. Bias and Mean Squared Error:

We study the approximate values as

$\bar{y} = \mu_Y(1+e_0)$

$\bar{x} = \mu_X(1+e_1)$

$\hat{\sigma}_Y^2 = \sigma_Y^2(1+e_2)$

$\hat{\sigma}_X^2 = \sigma_X^2(1+e_3)$

$\hat{\sigma}_{XY} = \sigma_{XY}(1+e_4)$

so that $E(e_0) = E(e_1) = E(e_2) = E(e_3) = E(e_4) = 0$

From Singh and Karpe (2009), we have

$E(e_0^2) = \frac{C_Y^2}{n\theta_Y}$ *and* $E(e_1^2) = \frac{C_X^2}{n\theta_X}$, where $\theta_Y = \frac{\sigma_Y^2}{\sigma_Y^2+\sigma_u^2}$ and $\theta_X = \frac{\sigma_X^2}{\sigma_X^2+\sigma_v^2}$

$$E(e_1e_3) = \frac{\mu_{3000}}{n\sigma_X^2\mu_X}$$

$$E(e_3^2) = \frac{A_X}{n}, where\ A_X = \gamma_{2X} + \gamma_{2v}\frac{\sigma_v^4}{\sigma_X^4} + 2\left(1+\frac{\sigma_v^2}{\sigma_X^2}\right)^2$$

$$E(e_0e_2) = \frac{\mu_{0300}}{n\sigma_Y^2\mu_Y}$$

$$E(e_0e_3) = \frac{\mu_{2100}}{n\sigma_X^2\mu_Y}$$

$$E(e_1e_2) = \frac{\mu_{1200}}{n\sigma_Y^2\mu_X}$$

$$E(e_0e_1) = \frac{\sigma_{XY}}{n\mu_X\mu_Y} = \frac{\rho C_X C_Y}{n}$$

$$E\left(e_2^2\right) = \frac{A_Y}{n}, where\ A_Y = \gamma_{2Y} + \gamma_{2u}\frac{\sigma_u^4}{\sigma_Y^4} + 2\left(1+\frac{\sigma_u^2}{\sigma_Y^2}\right)^2$$

$$E(e_2e_3) = \frac{\delta-1}{n}, where\ \delta = \frac{\mu_{2200}}{\sigma_X^2\sigma_Y^2}$$

$$E(e_1e_4) = \frac{\mu_{2100}}{n\sigma_{XY}\mu_X}$$

Now Expressing (1.1) in terms of e_i's, We have

$$\hat{\bar{y}}_{ME} = (\mu_Y + e_0\mu_Y)\exp\left\{k_1\left(\frac{\sigma_X^2(1+e_3)}{C_X^2} - \mu_X^2(1+e_1)^2\right)\right\}\exp\left\{k_2\left(\frac{\sigma_Y^2(1+e_2)}{C_Y^2} - \mu_Y^2(1+e_0)^2\right)\right\}$$

$$\hat{\bar{y}}_{ME} = (\mu_Y + e_0\mu_Y)\exp\left\{k_1\mu_X^2(1+e_3) - \mu_X^2(1+e_1)^2\right\}\exp\left\{k_2\mu_Y^2(1+e_2) - \mu_Y^2(1+e_0)^2\right\}$$

$$= (\mu_Y + e_0\mu_Y)\exp\left\{k_1\mu_X^2\left(e_3 - 2e_1 - e_1^2\right)\right\}\exp\left\{k_2\mu_Y^2\left(e_2 - 2e_0 - e_0^2\right)\right\}$$

On solving and approximating it to the first order, we have

$$\hat{\bar{y}}_{ME} = (\mu_Y + e_0\mu_Y)\left[1 + k_1\mu_X^2\left(e_3 - 2e_1 - e_1^2\right) + \frac{1}{2}k_1^2\mu_X^4\left(e_3^2 + 4e_1^2 - 4e_1e_3\right)\right]\left[1 + k_2\mu_Y^2\left(e_2 - 2e_0 - e_0^2\right) + \frac{1}{2}k_2^2\mu_Y^4\left(e_2^2 + 4e_0^2 - 4e_0e_2\right)\right]$$

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$$= \left(\mu_Y + e_0\mu_Y\right)\left\{1 + k_1\mu_X^2\left(e_3 - 2e_1\right) - k_1\mu_X^2 e_1^2 + k_2\mu_Y^2\left(e_2 - 2e_0\right) - k_2\mu_Y^2 e_0^2 + \frac{1}{2}k_1^2\mu_X^4\left(e_3^2 + 4e_1^2 - 4e_1e_3\right)\right.$$

$$\left. + \frac{1}{2}k_2^2\mu_Y^4\left(e_2^2 + 4e_0^2 - 4e_0e_2\right) + k_1k_2\mu_X^2\mu_Y^2\left(e_2e_3 - 2e_0e_3 - 2e_1e_2 + 4e_0e_1\right)\right\}$$

$$= \left(\mu_Y + \mu_Y\right)\left\{k_1\mu_X^2\left(e_3 - 2e_1\right) + k_2\mu_Y^2\left(e_2 - 2e_0\right) - k_1\mu_X^2 e_1^2 - k_2\mu_Y^2 e_0^2 + \frac{1}{2}k_1^2\mu_X^4\left(e_3^2 + 4e_1^2 - 4e_1e_3\right)\right.$$

$$\left. + \frac{1}{2}k_2^2\mu_Y^4\left(e_2^2 + 4e_0^2 - 4e_0e_2\right) + k_1k_2\mu_X^2\mu_Y^2\left(e_2e_3 - 2e_0e_3 - 2e_1e_2 + 4e_0e_1\right)\right\}$$

$$+ e_0\mu_Y + e_0\mu_Y\left\{k_1\mu_X^2\left(e_3 - 2e_1\right) + k_2\mu_Y^2\left(e_2 - 2e_0\right)\right\}$$

$$\hat{\bar{y}}_{ME} - \mu_Y = e_0\mu_Y + \mu_Y\left\{k_1\mu_X^2\left(e_3 - 2e_1\right) + k_2\mu_Y^2\left(e_2 - 2e_0\right)\right\} + \mu_Y\left[-k_1\mu_X^2 e_1^2 - k_2\mu_Y^2 e_0^2\right.$$

$$+ \frac{1}{2}k_1^2\mu_X^4\left(e_3^2 + 4e_1^2 - 4e_1e_3\right) + \frac{1}{2}k_2^2\mu_Y^4\left(e_2^2 + 4e_0^2 - 4e_0e_2\right)$$

$$+ k_1k_2\mu_X^2\mu_Y^2\left(e_2e_3 - 2e_0e_3 - 2e_1e_2 + 4e_0e_1\right)$$

$$\left. + k_1\mu_X^2\left(e_0e_3 - 2e_0e_1\right) + k_2\mu_Y^2\left(e_0e_2 - 2e_0^2\right)\right]$$

$$= \mu_Y\left[e_0 + k_1\mu_X^2\left(e_3 - 2e_1\right) + k_2\mu_Y^2\left(e_2 - 2e_0\right)\right] + \mu_Y\left\{\frac{1}{2}k_1^2\mu_X^4\left(e_3^2 + 4e_1^2 - 4e_1e_3\right)\right.$$

$$\left. + \frac{1}{2}k_2^2\mu_Y^4\left(e_2^2 + 4e_0^2 - 4e_0e_2\right) - k_1\mu_X^2 e_1^2 - k_2\mu_Y^2 e_0^2\right\}$$

$$+ k_1\mu_X^2\left(e_0e_3 - 2e_0e_1\right) + k_2\mu_Y^2\left(e_0e_2 - 2e_0^2\right)$$

$$+ k_1k_2\mu_X^2\mu_Y^2\left(e_2e_3 - 2e_0e_3 - 2e_1e_2 + 4e_0e_1\right)$$

$$\hat{\bar{y}}_{ME} - \mu_Y = \mu_Y\left(e_0 + \mu_X^2k_1e_3 - 2\mu_X^2k_1e_1 + \mu_Y^2k_2e_2 - 2\mu_Y^2k_2e_0\right)$$

$$+ \mu_Y\left\{\frac{k_1^2}{2}u_X^4\left(e_3^2 + 4e_1^2 - 4e_1e_3\right) + \frac{k_2^2}{2}\mu_Y^4\left(e_2^2 + 4e_0^2 - 4e_0e_2\right)\right.$$

$$\left. - k_1\mu_X^2\left(e_1^2 - e_0e_3 + 2e_0e_1\right) - k_2\mu_Y^2\left(e_0^2 - e_0e_2 + 2e_0^2\right)\right\}$$

$$+ k_1k_2\mu_X^2\mu_Y^2\left(e_2e_3 - 2e_0e_3 - 2e_1e_2 + 4e_0e_1\right)$$

$$\hat{\bar{y}}_{ME} - \mu_Y = \mu_Y\left(e_0 + \mu_X^2k_1e_3 - 2\mu_X^2k_1e_1 + \mu_Y^2k_2e_2 - 2\mu_Y^2k_2e_0\right)$$

$$+ 2\mu_Y\left\{k_1^2u_X^4\left(e_3^2 + 4e_1^2 - 4e_1e_3\right) + k_2^2\mu_Y^4\left(e_2^2 + 4e_0^2 - 4e_0e_2\right)\right.$$

$$\left. - 2k_1\mu_X^2\left(e_1^2 - e_0e_3 + 2e_0e_1\right) - 2k_2\mu_Y^2\left(e_0^2 - e_0e_2 + 2e_0^2\right)\right\}$$

$$+ 2k_1k_2\mu_X^2\mu_Y^2\left(e_2e_3 - 2e_0e_3 - 2e_1e_2 + 4e_0e_1\right) \qquad (2.1)$$

Taking expectation on both the sides of (2.1), the bias is given by,

$$Bias\left(\hat{\hat{y}}_{ME}\right) = E\left(\hat{\hat{y}}_{ME} - \mu_Y\right) = 2\mu_Y \left\{ k_1 \mu_X^4 \left(\frac{A_X}{n} + 4\frac{C_X^2}{n\theta x} - 4\frac{\mu_{3000}}{n\sigma_X^2 \mu_x} \right) + k_2^2 \mu_Y^4 \left(\frac{A_Y}{n} + 4\frac{C_Y^2}{n\theta_Y} - 4\frac{\mu_{0300}}{n\sigma_Y^2 \mu_Y} \right) \right.$$

$$- 2k_1 \mu_X^2 \left(\frac{C_X^2}{n\theta_X} - \frac{\mu_{2100}}{n\sigma_X^2 \mu_Y} + 2\rho \frac{C_X C_Y}{n} \right) - 2k_2 \mu_Y^2 \left(\frac{C_Y^2}{n\theta_Y} - \frac{\mu_{0300}}{n\sigma_Y^2 \mu_Y} + 2\frac{C_Y^2}{n\theta_Y} \right)$$

$$\left. + 2k_1 k_2 \mu_X^2 \mu_Y^2 \left(\frac{\delta - 1}{n} - 2\frac{\mu_{2100}}{n\sigma_X^2 \mu_Y} - 2\frac{\mu_{1200}}{n\sigma_Y^2 \mu_Y} + 4\rho \frac{C_X C_Y}{n} \right) \right\}$$

(2.2)

Now on squaring both sides (2.1), we get

$$\left(\hat{\hat{y}}_{ME} - \mu_Y\right)^2 = \mu_Y^2 \left\{ e_0 + \mu_X^2 k_1 (e_3 - 2e_1) + \mu_Y^2 k_2 (e_2 - 2e_0) \right\}^2$$

$$= \mu_Y^2 \{$$

$$e_0^2 + \mu_X^4 k_1^2 \left(e_3^2 + 4e_1^2 - 4e_1 e_3\right) + \mu_Y^4 k_2^2 \left(e_2^2 + 4e_0^2 - 4e_0 e_2\right) + 2\mu_X^2 k_1 (e_0 e_3 - 2e_0 e_1)$$

$$\left. + 2\mu_Y^2 k_2 \left(e_0 e_2 - 2e_0^2\right) + 2\mu_X^2 \mu_Y^2 k_1 k_2 (e_3 e_2 - 2e_0 e_3 - 2e_1 e_2 + 4e_0 e_1) \right\}$$

(2.3)

Considering the expectations on both sides of (2.3) and then the mse is given by,

$$MSE\left(\hat{\hat{y}}_{ME}\right) = E\left(\hat{\hat{y}}_{ME} - \mu_Y\right)^2 = \mu_Y^2 \frac{C_Y^2}{n\theta_Y} + \mu_Y^2 \left\{ \mu_X^4 k_1^2 \left(\frac{A_X}{n} + 4\frac{C_X^2}{n\theta_X} - 4\frac{\mu_{3000}}{n\sigma_X^2 \mu_X} \right) \right.$$

$$+ \mu_Y^4 k_2^2 \left(\frac{A_Y}{n} + 4\frac{C_Y^2}{n\theta_Y} - 4\frac{\mu_{0300}}{n\sigma_Y^2 \mu_Y} \right)$$

$$+ 2\mu_X^2 k_1 \left(\frac{\mu_{2100}}{n\sigma_X^2 \mu_Y} - 2\rho \frac{C_X C_Y}{n} \right) + 2\mu_Y^2 k_{2.} \left(\frac{\mu_{0300}}{n\sigma_Y^2 \mu_Y} - 2\frac{C_Y^2}{n\theta_Y} \right)$$

$$\left. + 2\mu_X^2 \mu_Y^2 k_1 k_2 \left(\frac{\delta - 1}{n} - 2\frac{\mu_{2100}}{n\sigma_X^2 \mu_Y} - 2\frac{\mu_{1200}}{n\sigma_Y^2 \mu_X} + 4\rho \frac{C_X C_Y}{n} \right) \right\}$$

$$MSE\left(\hat{\hat{y}}\right)_{ME} = \mu_Y^2 \frac{C_Y^2}{n\theta_Y} + \left[\mu_Y^2 k_1^2 \delta_{11} + k_2^2 \delta_{22} + 2k_1 \delta_{10} + 2k_2 \delta_{02} + 2k_1 k_2 \delta_{12} \right]$$

(2.4)

where $\delta_{11} = \mu_X^4 \left(\frac{A_X}{n} + 4\frac{C_X^2}{n\theta_X} - 4\frac{\mu_{3000}}{n\sigma_X^2 \mu_X} \right)$

$$\delta_{22} = \mu_Y^4 \left(\frac{A_Y}{n} + 4\frac{C_Y^2}{n\theta_Y} - 4\frac{\mu_{0300}}{n\sigma_Y^2 \mu_Y} \right)$$

$$\delta_{10} = \left\{ \mu_X^2 \left(\frac{\mu_{2100}}{n\sigma_X^2 \mu_Y} - 2\rho \frac{C_X C_Y}{n} \right) \right\}$$

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$$\delta_{02} = \left\{ \mu_Y^2 \left(\frac{\mu_{0300}}{n\sigma_Y^2 \mu_Y} - 2\frac{C_Y^2}{n\theta_Y} \right) \right\}$$

$$\delta_{12} = \left\{ \mu_x^2 \mu_Y^2 \left(\frac{\delta - 1}{n} - 2\frac{\mu_{2100}}{n\sigma_X^2 \mu_Y} - 2\frac{\mu_{1200}}{n\sigma_Y^2 \mu_X} + 4\rho\frac{C_X C_Y}{n} \right) \right\}$$

For optimizing (2.4) w.r.t k_1& k_2, we have the two normal equations as

$$\delta_{11}k_1 + \delta_{12}k_2 + \delta_{10} = 0 \tag{2.5}$$

$$\delta_{12}k_1 + \delta_{22}k_2 + \delta_{02} = 0 \quad . \tag{2.6}$$

On solving these two normal equations for k_1 & k_2, the optimum values of k_1& $k_2$are given by

$$k_1 = \frac{\delta_{22}\delta_{10} - \delta_{02}\delta_{12}}{\delta_{12}^2 - \delta_{11}\delta_{22}} \tag{2.7}$$

$$k_2 = \frac{\delta_{11}\delta_{02} - \delta_{12}\delta_{10}}{\delta_{12}^2 - \delta_{11}\delta_{22}} \tag{2.8}$$

For these optimum values of k_1 and k_2 the minimum mean squared error of $\hat{\hat{y}}_{ME}$is given by

$$MSE\left(\hat{\hat{y}}_{ME}\right)_{\min} = \left(\frac{\mu_Y^2 C_Y^2}{n\theta_Y} \right) - \frac{\mu_Y^2 \left(\delta_{11}\delta_{02}^2 + \delta_{22}\delta_{10}^2 - 2\delta_{02}\delta_{10}\delta_{12} \right)}{\left(\delta_{11}\delta_{22} - \delta_{12}^2 \right)} \tag{2.9}$$

3.Theoretical Comparison

We compare the proposed estimator's MSE with the usual mean per unit estimator that is.

$$\bar{y}_m = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{3.1}$$

$$\bar{y}_m = \mu_Y (1 + e_0)$$

Expressing (1.1) in terms of e_i's, $\bar{y}_m$ becomes

$$\bar{y}_m = \mu_Y (1 + e_0)$$

$$\bar{y}_m - \mu_Y = \mu_Y e_0$$

Therefore $Bias(\bar{y}_m) = 0$ (3.2)

$$MSE(\bar{y}_m) = \frac{\sigma_Y^2}{n}\left(1 + \frac{\sigma_X^2}{\sigma_Y^2} \right) \tag{3.3}$$

Now, the proposed estimator $\hat{\hat{y}}_{ME}$ will be more efficient than the usual mean per unit estimator in presence of measurement error if

$$MSE(\bar{y}_m) - MSE\left(\hat{\hat{y}}_{ME}\right) > 0$$

or $\frac{1}{2\sigma_Y^2}\left(\frac{\mu_Y^2 \sigma_X^2}{\theta_Y} + \frac{\sigma_{XY}^2}{\sigma_X^2 \theta_Y} - \sigma_Y^2 - \sigma_X^2 - h \right) < \rho^2$

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or $h > \frac{1}{2\sigma_Y^2}\left(\frac{\mu_Y^2\sigma_X^2}{\theta_Y} + \frac{\sigma_{XY}^2}{\sigma_X^2\theta_Y} - \sigma_Y^2 - \sigma_X^2\right) - \rho^2$, where $h = \frac{\left(\delta_{11}\delta_{02}^2 + \delta_{22}\delta_{10}^2 - 2\delta_{02}\delta_{10}\delta_{12}\right)}{\left(\delta_{11}\delta_{22} - \delta_{12}^2\right)}$. (3.4)

Hence the proposed estimator $\hat{\hat{y}}_{ME}$ will be more efficient than the usual mean per unit estimator in presence of measurement error if the condition (3.4) is satisfied by the data set.

4. Empirical Study

Gujrati and Sangeeta's (2007) page 539 has taken data statistics used in the empirical study where

Y= Accurate Consumption Expenses
X= True Earnings
y_i= Measured consumption expenses
x_i = Calculate income
$n = 10,$
$\bar{X} = 170,$
$\bar{Y} = 127,$
$\sigma_X^2 = 3300,$
$\sigma_Y^2 = 1278,$
$\sigma_u^2 = 32.4001,$
$\sigma_v^2 = 32.3998$
$C_Y = 0.2815,$
$C_X = 0.3379,$
$\rho_{XY} = 0.9641,$
$\beta_{2Y} = 1.9026,$
$\beta_{2X} = 1.7758,$
$\beta_{2u} = 1.7186,$
$\beta_{2v} = 1.8409$

The computed MSEs of the estimators with measurement errors are provided by
$MSE(\bar{y}_m) = 131.083$
$MSE(\bar{y}_m) = 14.969.$

5. Conclusion

A proposed estimator's performance $\hat{\hat{y}}_{ME}$ is judged by using the mean square error criterion to evaluate the effectiveness of the estimators has allowed for the theoretical and empirical establishment of the presence of measurement errors. By theoretical comparison it is discovered that the proposed estimate performs better in terms of MSE when compared to the mean per unit estimator. The relative efficiency (PRE) of the proposed estimator over the mean per unit estimator under measurement error is calculated using the above MSEs is 875 showing the enhanced efficiency of the proposed estimator

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