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Isolated Signed Total Dominating Function of Directed Graphs

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Abstract
An isolated sighed total dominating function (ISTDF) of a digraph is a function f:$V(D)$⟶$\{-1,+1\}$ such that $\sum_{u\in N}-(v)\geq 1$ for every vertex $v\in V(D)$and for at least one vertex of $w\in V(D)$,$f\left(N^{-}(w)\right)=+1$. An isolated signed total domination number of D, denoted by $\gamma_{ist}(D)$, in the minimum weight of an isolated signed total dominating function of D. In this paper, we study some properties of ISTDF
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Key Words: isolated domination, signed dominating function, isolated signed dominating function, isolated signed total dominating function.

1. Introduction

In this paper, we consider $D=(V((D),A(D))$ be a digraph with p vertices and q arcs. For a vertex $v\in V(D)$, the set $I(v)=\{u:(u,v)\in A(D)\}$ is called the in-neighborhood of v. The in-degree of u is defined by $deg^{-}(v)=|I(v)|$. A general reference for graph theoretic notions is [3].

For any $v\in(V(D),N^{+}(v)=\{u\in V(D)|v$ is adjacent to $u\}$ and $N^{+}[v]=N^{+}(v)\cup\{v\}$ are called open out-neighborhood and closed out-neighborhood of v respectively. Similarly $N^{-}(v)=\{u\in(V(D)|is\ adjacent\ from\ u\}$ and $N^{-}[v]=N^{-}(v)\cup\{v\}$ are called open in neighborhood and closed in-neighborhood respectively.

A function $f:V(G)\rightarrow\{0,1\}$ is called a dominating function if for every vertex $v\in V(G),f(N[v]\geq 1$ [6]. The weight of f, denoted by $w(f)$ is the sum of the values $f(v)$ for all $v\in V(G)$.

Various domination functions has been defined from the definition of dominating function by replacing the domain $\{0,1\}$, as one of the sets $\{-1,0\},\{-1,+1\}$ and etc. One of such example in signed dominating function [5,4].

A subset S of vertices of a graph is a total dominating set of G if every vertex in $V(G)$ has a neighbor in S. The minimum cardinality of a total dominating set of G is said to be the total dominating set of G is said to be the total domination number and is denoted by $\gamma_t(G)$.

The Ciência & Engenharia - Science & Engineering Journal
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Volume 10 Issue 1, 2022
pp: 265 - 271

In 1995, J. E. Dunbar et al [4] defined signed dominating function of an undirected graph. A function $f: V(G) \rightarrow \{-1, +1\}$ is a signed dominating function of G, if for every vertex $v \in V(G), f(N[v]) \geq 1$. The signed domination number, denoted n by $\gamma_s(G)$, is the minimum weight of a signed dominating function on G [4]

In 2001, Bohadan Zelinka and Liberec [1], by using the definition total domination and signed domination and introduced the concept of signed total dominationnumber of a graph. A mapping $f: V(D) \rightarrow \{-1, +1\}$, where $V(G)$ is the vertex set of G, is called a signed total dominating function (STDF) on G, if $\sum_{x \in N(v)} f(x) \geq 1$ for each $v \in V(G)$. The minimum of values $\sum_{x \in N(v)} f(x) \geq 1$ taken over all STDF's of G, is called the signed total domination number of G and denoted by $\gamma_{st}(G)$.

In 2019, S. Rishitha Dayana and S Chandra Kumar [10], by using the definitiion of signed dominating function an isolate domnination, defined the new domination parameter called isolated signed dominating function. An isolated signed dominating function (ISDF) of a graph G is a SDF function such that (N) $f(N[w]) = +1$ for at leat one vertex of $w \in V(G)$. The weight f, denoted by $w(f)$ is the sum of the value for all $v \in V(G)$. An indated signed domination number of G , denoted by $\gamma_{is}(G)$.is the minimum weight of an ISDF of G.

In 2021, S. Sunitha and S. Chandra Kumar, by using the definition of signed total dominating function and isolate domination and introduced the concept of dates isolated signed total dominating function . An isolated signed total dominating function (1STDF) of a graph G is a function $f: V(G) \rightarrow \{-1, +1\}$ such that $\sum_{v \in N(v)} f(v) \geq 1$ for every vertex $v \in V(G)$. and for at least one vertex $w \in V(G)$, $f(N(w)) = +1$ An isolated signed total domination number of C denoted (C), is the minimum weight of added signed total dominating function of G .

In this paper, we study an isolated signed total domiinating Function (ISTDF) of a digraph. An ISTDF of a digraph D is a function $f: V(D) \rightarrow \{-1, +1\}$ suchthat $\sum_{u \in N^-(u)} f(u) \geq 1$ for every vertex $v \in V(D)$ and for atleast one vertex of $w \in V(D)$, $f(N^-(w)) = +1$. An isolated signed total domination number of D, denoted by $\gamma_{ist}(D)$, is the minimum weight of an isolated signed total dominating function of D. In this paper, we study some properties of ISTDF and we give isolated signed total domination number some classes of graphs.

2. Main Results

Lemma 1. Let D be any digraph in which indegree of v is even for all $v \epsilon V(D)$. Then D does not admit ISTDF.

Proof :

Note that $|N^-(u)|$ is even for any vertex $u \epsilon V(D)$.

Thus there exist no vertex $v \epsilon V(D)$ such that $f(N^-(u)) = 1$ for any function $f : V(D) \rightarrow$ {-1,+1}.

Lemma 2

For any digraph D which admits ISTDF, γ_{st} (D) ≤ γ_{ist} (D).

The Ciência & Engenharia - Science & Engineering Journal
ISSN: 0103-944X
Volume 10 Issue 1, 2022
pp: 265 - 271

Proof :

Since every ISTDF is a STDF, it follows that γ_{st} (D) ≤ γ_{ist} (D).

Theorem 3

Let $n \geq 2$ be an integer and let D be a disconnected digraph with n components D_1, D_2,............D_n such that the first $r(\geq 1)$ components D_1, D_2,............D_r admit ISTDF.

Then γ_{ist} (D) = $\min_{1\leq i\leq r}\{t_i\}$, where $t_i = \gamma_{ist}(D_i) + \sum_{j=1,j\neq i}^{n} \gamma_{ist}(D_j)$.

Proof :

Assume that $t_1 = \min_{1\leq i\leq r}\{t_i\}$.

Let f_1 be an minimum ISTDF of $D_1$and f_i be a minimum STDF of D_i for each i with 2 $\leq i \leq n$.

Then $f : V(D) \to \{-1,+1\}$ defined by $f(x) = f_i(x)$, $x \in V(D_i)$, is an ISTDF of D with weight $\gamma_{ist}(D_1) + \sum_{i=2}^{n} \gamma_{ist}(D_i) = t_1$.

Let g be a minimum ISTDF of D.

Then there exists an integer j such that $g|D_j$ for some j with $1 \leq j \leq r$.

Also for each i with $1\leq i \leq n(i \neq j)$, $g|D_i$, is a minimum STDF of D_i.

Therefore $w(g) \geq \gamma_{ist}(D_j) + \sum_{i=1,i\neq j}^{n} \gamma_{ist}(D_i) = t_j \geq t_1$ and hence $\gamma_{ist}(D) = \min_{1\leq i\leq r}\{t_i\}$.

Corollary 4

Let H be any digraph which does not admit ISTDF.

Then D = H ∪ r $\overrightarrow{C_3}$ (r ≥ 1) admits ISTDF with

$\gamma_{ist}(D)$ = 3r + $\gamma_{st}(H)$.

Proof :

Let $D_i \cong C_3$ for $1\leq i \leq r$ and $D_{r+1} \cong H$.

Note every vertex of each copy of $\overrightarrow{C_3}$ receives the label +1.

Thus by theorem 4.1.1., we have $\gamma_{ist}(D)$ = 3r + $\gamma_{st}(H)$.

Lemma 5

Let f be an ISTDF of D and let $S \subset V$.

Then $f(S) \equiv |S|(\text{mod } 2)$.

Proof :

Let $S^+ = \{v \mid f(v) = 1, v \in S\}$ and $S^- = \{v \mid f(v) = -1, v \in S\}$.

Then $|S^+| + |S^-| = |S|$ and $|S^+| - |S^-| = f(S)$.

If both S^- and S^+ are either odd or even, then both $|S|$ and $f(S)$ must be even.

If either one of S^- and S^+ is odd and another one is even, then both $|S|$ and $f(S)$ must be odd.

Therefore $f(S) \equiv |S|(\text{mod}2)$.

Lemma 6

Let D be a digraph of order n and $\delta^- \geq 2$.

Then $2_{y2,t}$(D) - $n \leq \gamma_{ist}(D)$.

Proof :

Let g be a minimum isolate signed total dominating function of D.

Let $V^+ = \{u \in V : g(u) = +1\}$ and $V^- = \{v \in V : g(v) = -1\}$.

The Ciência & Engenharia - Science & Engineering Journal
ISSN: 0103-944X
Volume 10 Issue 1, 2022
pp: 265 - 271

If V^- = $\emptyset$, then the proof is clear.

Suppose there exists a vertex v V^-.

Since $g(N^-(v)$ ≥ 1 and δ^- ≥ 2, then v has at least two adjacent vertices in V^+.

In the similarly manner, if v V^+, then v has at least two adjacent vertices in V^+.

Therefore V^+ is a 2- total dominating set for D and so $|V^+|$ ≥ $\gamma_{2,t}$(D).

Since $\gamma_{ist}(D)$ = $|V^+|$ - $|V^-|$ and n = $|V^+|$ + $|V^-|$, we have $\gamma_{ist}(D)$ = 2$|V^+|$ - n and so $\gamma_{ist}(D)$ ≥ 2$\gamma_{2,t}$(G)- n.

Remark 7

(a) Let D be a digraph which admits a 2- total dominating set S.

Then $N^-(v) \subseteq S$ whenever $|N^-(v)|$ = 2 for any vertex $v \epsilon$ V (D).

(a) Let D be a digraph which admits an ISTDF function (or STDF), say *f*.

Then the vertices of $N^-(v)$ are labelled with +1 sign whenever

$|N^-(v)|$ ≤ 2 for any vertex $v \epsilon$ V (D).

Theorem 8

For given integer k ≥ 1, there exists a digraph D such that $\gamma_{st}(D)$ = $\gamma_{ist}(D)$ = k.

Proof :

Let D be a digraph such that V(D) = $\{a_1, a_2, ..., a_{2k}, b_2, b_4, b_6, ..., b_{2k}\}$ and A(D) = $\{(a_i, a_{i+1})$/ 1 ≤ i ≤ 2k -1} ∪ $\{(a_{2i}, b_{2i})$: 1 ≤ i ≤ $k\}$.

Let *f* be a ISTDF of D.

Then by Remark 4.1.1.(b), *f*(a_i) = +1 for all i with 1 ≤ i ≤ 2k.

Thus *f*(V(D)) ≥ 2k (+1) + k(-1) = k and so $\gamma_{st}(D)$ ≥ k.

Define a function g : V(D) → {-1, +1} by g (a_i) = +1 and g (b_i) = -1.

Then g is a STDF such that w (*f*) = k and *f*($N^-(b_2)$) = 1.

Therefore $\gamma_{ist}(D)$ ≤ k.

Since $\gamma_{st}(D)$ ≤ $\gamma_{ist}(D)$, we have $\gamma_{st}(D)$ = $\gamma_{ist}(D)$ = k.

Lemma 9

If G = $m\overleftrightarrow{P_2}$ ∪ B, where B is a graph which is an union of cycles (m≥ 1 and B may be empty), then $\gamma_{ist}(D)$ = n, where n is the order of D.

Proof :

Let *f* be an ISTDF of D and $u \epsilon$ *V(D).*

Case 1 :

If $u \epsilon$ $V(m\overleftrightarrow{P_2})$, then by Remark 4.1.1(b), *f*(u) = +1.

Case 2 :

If $u \epsilon$ V (B).

Then u must be in an unidirectional cycle, say B_1.

Let $V(B_1)$ = {v1, v_2,......., v_n} and $A(B_1)$ = {(v_1, v_2), (v_2, v_3),......., (u_{n-1}, u_n), (u_n, v_1).

Subcase 1 :

If $u \epsilon$ $V(B_1)$ and u = v_i for some i with 1 ≤ i ≤ n -1.

The Ciência & Engenharia - Science & Engineering Journal
ISSN: 0103-944X
Volume 10 Issue 1, 2022
pp: 265 - 271

Since $f(N^-(v_{i+1})) \geq 1$, we have $f(v_i) = f(u) = +1$.

Subcase 2 :

If $u \epsilon V(B_1)$ and $u = u_n$.

Since $f(N^-(v_1)) \geq 1$, we have $f(v_n) = f(u) = +1$.

Thus $w(f) = n$ and so $\gamma_{ist}(D) \geq n$.

But always so $\gamma_{ist}(D) \leq n$ and so $\gamma_{ist}(D) = n$.

Remark 10

Let *G* be a digraph of order *n* which admits ISTDF.

Then $\gamma_{ist}(D) \neq n$-1.

Proof :

Let *f* be a minimum ISTDF of *D*.

Suppose $f(u) = -1$ for some $u \epsilon V(D)$, then $\gamma_{ist}(D) \leq n - 2$.

Lemma 11

Let *D* be a digraph of order *n*.

If $1 \leq n \leq 2$, then $\gamma_{ist}(D) = n$, and if $n \geq 3$, then $4 - n \leq \gamma_{ist}(D) \leq n$.

Proof :

If $n = 1$, *D* is isomorphic to K_1.

If $n = 2$, *D* is isomorphic to $\overrightarrow{P_2}$ or $2K_1$ or $\overleftrightarrow{P_2}$.

When *D* is isomorphic to $\overrightarrow{P_2}$ or $2K_1$, *D* does not admit ISTDF.

In the other case, we have to label all the vertices with +1 sign for any STDF.

Thus $\gamma_{ist}(D) = n$, when $n = 1$ or 2.

Let $n \geq 3$.

The upper bound $\gamma_{ist}(D) \leq n$ is immediate.

If *f* is a signed total dominating function on *D*, then the condition $n \geq 3$ implies that there are at least two distinct vertices *u* and *v* such that $f(u) = f(v) = 1$, and thus $\gamma_{ist}(D) \geq 2(+1) - (n-2)(-1) = 4 - n$.

Theorem 12

Let *q* be a positive integer.

Then there exists a digraph *D* with $q + 8$ vertices such that *D* has a Hamiltonian directed cycle and such that $\gamma_{ist}(D) = -q + 2$.

Proof :

Let $V(D) = \{u_1, u_2, u_3, u_4, v_1, v_2, \ldots\ldots. v_{q+4}\}$.

Consider the directed cycle *H* such that $V(H) = V(D)$ an the arcs of *H* are (u_1, u_2), (u_2, u_3), (u_3, u_4), (u_4, v_1), (v_1, v_2),.......$(v_{q+3}, v_{q+4}$, (v_{q+4}, u_1).

From the cycle H we construct the digraph D by adding the edge (u_4, u_1) and the edges (u_2, v_i), (u_3, v_i) for $i \epsilon \{1,\ldots\ldots., q + 4\}$, and (u_4, v_i) for $i \epsilon \{2,\ldots\ldots., q + 4\}$.

Let $f : V(D) \rightarrow \{-1,1\}$ be such that $f(u_i) = +1$ for $i \epsilon \{1,2,3,4\}$ and $f(v_i) = -1$ for $i \epsilon \{1,\ldots\ldots., q + 3\}$ and $f(v_{q+4}) = +1$.

Thus $f(N^-(v_i)) = f(u_4) + f(v_{q+4}) = 1 + 1 = 2$.

For $i = 2,3,4$, we have $f(N^-(u_i)) = f(N^-(u_{i-1})) = 1$.

Also $f(N^-(v_1)) = f(u_2) + f(u_3) + f(u_4) = 1 + 1 + 1 = 3$.

The Ciência & Engenharia - Science & Engineering Journal
ISSN: 0103-944X
Volume 10 Issue 1, 2022
pp: 265 - 271

For i = 2,3,..........,q + 4, we have $f(N^{-}(v_i))$ = $f(N^{-}(v_{i-1}))$ + $f(u_2)$ + $f(u_3)$ + $f(u_4)$ = (-1) + 1 + 1 + 1 = 2.

Thus f is a ISTDF and $w(f)$ = 4(+1) + (q +3) (-1) + 1(+1) = 4 – q – 3 + 1 = - q +2.

Hence $\gamma_{ist}(D) \leq$ - q +2.

By Remark 4.1.1(b), $f(u_i)$ = + 1 for $1 \leq i \leq 4$ and $f(v_{q+4})$ = +1.

Thus $\gamma_{ist}(D) \geq$ 5 (+1) + (q+3) (-1) = 5+ -q -3 = -q + 2.

Definition 13

The bidirectional cycle $\overleftrightarrow{C_n}$ is defined as a digraph with $V(\overleftrightarrow{C_n}) = \{v_1, v_2, ..., v_n\}$ and $A(\overleftrightarrow{C_n}) = \{a_i = (v_i, v_{i+1}) : 1 \leq i \leq n\} \cup \{b_j = (v_j, v_{j+1}) : 1 \leq i \leq n\}$, where the subscript is taken modulo n.

Definition 14

The unidirectional path $\overleftrightarrow{P_n}$ is defined as a digraph with V $(\overleftrightarrow{P_n}) = \{v_1, v_2, ..., v_n\}$ and $A(\overleftrightarrow{P_n}) = \{a_i = (v_i, v_{i+1}) : 1 \leq i \leq n - 1\}$.

Lemma 15

(a) The bidirectional cycle $\overleftrightarrow{C_n}$ of order $n \geq 3$ does not admit ISTDF.

(b) The path $\overleftrightarrow{P_n}$ of order $n \geq 2$ does not admit ISTDF.

Proof :

(a) Let f be a ISTDF of $\overleftrightarrow{C_n}$.

Note that $|N^{-}(v)| = 2$ for all $v \in$ V$(\overleftrightarrow{C_n})$.

By Remark 4.1.1(b) all the vertices of $\overleftrightarrow{C_n}$ must have +1 sign.

In this case $f(N^{-}(v)) = 2$ for all $v \in$ V(C_n), a contradiction.

(b) Let g be a ISTDF of $\overleftrightarrow{P_n}$.

Note that $|N^{-}(v_1)| = 0$, a contradiction.

Definition 16

The unidirectional cycle $\overleftrightarrow{C_n}$ is defined as a digraph with V $(\overleftrightarrow{C_n}) = \{v_1, v_2, ..., v_n\}$ and $A(\overleftrightarrow{C_n}) = \{a_i = (v_i, v_{i+1}) : 1 \leq i \leq n\}$, where the subscript is taken modulo n.

Lemma 17

Let $n \geq 3$ be an integer.

Then the unidirectional cycle $\overleftrightarrow{C_n}$ admits ISTDF with $\gamma_{ist}(\overleftrightarrow{C_n}) = n$.

Proof :

Let f be a ISTDF of $\overleftrightarrow{C_n}$.

Note that $N^{-}(v_i) = \{v_{i-1}\}$ and $N^{-}(v_i)| = 1$ for $1 \leq i \leq n$.

By Remark 4.1.1(b) all the vertices of $\overleftrightarrow{P_n}$ must have + 1 sign.

In this case $f(N^{-}(v_i)) = 1$ for 1 for $1 \leq i \leq n$.

Thus $\gamma_{ist}(\overleftrightarrow{C_n}) \leq n$.

The Ciência & Engenharia - Science & Engineering Journal
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Volume 10 Issue 1, 2022
pp: 265 - 271

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