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Optimal Design of Two Junction Four ReservoirWater Transmission System

[1] A. R. Bansod, [2] R. S. Tatwawadi
[1]1Assistant Professor, Department of Civil Engineering, Jawaharlal Darda institute of Engineering &Technology, Yavatmal, Maharashtra, India,
[2] Principal, Jawaharlal Darda institute of Engineering &Technology, Yavatmal, Maharashtra, India
[1]amol_bansod@jdiet.ac.in, [2]principal@jdiet.ac.in

Abstract: Applying mathematical models to obtain optimal solution for water transmission system has been of major concern during the last few decades for optimal utilization of public fund which is limited. Optimization problem for water transmission system is considered as a non-linear problem during the search process to identify the decision variables.In this paper, flow path algorithm coupled with dynamic programming has been adopted to obtain the optimal design of the multi reservoirwater transmission system.

Index Terms: Dynamic programming,Flow path algorithm, Lagrange multiplier method, Optimization, Water transmission system

I. Introduction

Raw water from various sources such as river is taken to water treatment plant for purification and treatment so as to make the same suitable for drinking purpose. The pipeline conveying water from the source of water to the water treatment plant, and from the water treatment plant to the break pressure tank is known as rising main or pumping main. The pipeline conveying water from break pressure tank to the main master balancing reservoir is known as gravity main. The pipeline conveying water from main master balancing reservoir to the distribution master balancing reservoir is known as trunk transmission line. The pipeline from the distribution master balancing reservoir to the elevated service reservoir is known as zonal transmission line. The trunk transmission lines and the zonal transmission lines are commonly known as the water transmission lines. The proposed work deals with the design of such water transmission lines. The design of water transmission line requires large diameter pipe along with its accessories such as air relief valves, scour valves and sluice valves etc., and these pipes as well as accessories involves huge cost. Therefore, there is a need for the optimal design of such water transmission lines for multi reservoir multi junction system. Thus, a substantial reduction in the overall cost of the water transmission lines can be achieved. This paper deals with the optimal design of a small water transmission system involving two junctions. The design of water transmission line requires large diameter pipe along with its accessories such as air relief valves, pressure relief valves, scour valve, sluice valve etc., and these pipes as well as accessories involves huge construction as well as operation and maintenance cost. Therefore, there is a need to

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obtain the optimal design of such water transmission systems so as to obtain substantial reduction in the overall cost of the water transmission lines.A simple algorithm for the optimal design of a water transmission system has been developed that can be used to determine optimal diameter, and optimal cost of the water transmission pipelines. In this paper,a simple procedure for the optimal design of two junction water transmissionsystemusing Hazen-Williams equation has been developed.

II. Review of Literature

Shamir [1] developed a methodology for the optimal design and operation of a water distribution system that operates under one or several loading conditions. It was concluded that the method was computationally feasible.

Bhave [2] developed a method for the optimal expansion of water distribution systems subjected to a single loading pattern. The method was iterative and converges rapidly to a local but fairly good optimal solution. Optimization of an entirely new distribution system is a particular characteristic of this general approach.

Fujiwara and Dey [3] proposed a computationally efficient two-stage method for the optimal design of branched water distribution networks on flat terrain. In the initial stage, the Lagrange multiplier method was used to obtain continuous optimal pipe sizes and Lagrange multipliers analytically. It was not necessary here to solve a system of linear equations and only simple arithmetic operations were needed. For any method in which continuous diameters were evaluated, a heuristic procedure is used to round off the continuous diameter into discrete diameters.

Sharma and Swamee [4] developed a method for flow path identification. A node (nodal point) receives water through various paths. These paths are called flow paths. Knowing the discharges flowing in the pipe links, the flow paths can be obtained by starting from a node and proceeding in a direction opposite to the flow.

Sarbu [5] studied the optimization of water distribution networks supplied from one or more node sources, according to demand variation. The work treats looped networks which have concentrated outflows or uniform outflow along the length of each pipe. An improved model was developed for the optimal design of new or partially extended water distribution networks, which operate either by means of gravity or pumping system. The model was based on the method of linear programming and allows the determination of an optimal distribution of commercial diameters for each pipe in the network and the length of the pipes which correspond to these diameters.

Sarbu and Valea [6] suggested that determination of pipe network optimal path is important for an effective modelling and optimization of water distribution systems. The principal application of the branched network optimal path

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was to evaluate the hydraulic performance of the distribution system based on the selected schemes for many types of network analysis (e.g. design, operation, and calibration). Already known algorithms for solving this problem usually determine a sole solution which in some cases proves to be suboptimal.

Chiplunkar and Khanna [7] developed a rational procedure for the optimization of branched water supply networks that yields closed form solution. Two case studies were presented to illustrate the procedure for the optimal design of complete gravity systems and direct pumping systems.

Lansey and Mays [8] developed a new methodology for determining the optimal (minimum cost) design of water distribution systems. The components that can be sized were the pipe network, pumps or pumping station, and tanks. In addition, the optimal settings for control and pressure-reducing valves could be determined. This methodology couples nonlinear programming techniques with existing water distribution simulation models. Previous methodologies have typically simplified the system hydraulics to be able to solve the optimization problem. The new methodology retains the generality of the hydraulic simulation model so that the problem was only limited by the ability of the simulation model rather than the optimization model. The methodology used a generalized reduced gradient model to solve a problem that is reduced in size and complexity by implicitly solving the conservation of mass and energy equations using the hydraulic simulator and an augmented Lagrangian approach to incorporate pressure head bounds in the objective function.

III. Hydraulic Equation

The Hazen-Williams equation relating velocity of flow, hydraulic mean radius and hydraulic gradient is given by:

$V = 0.849 CH\, R^{0.63}\, S^{0.54}$ (1)

Where, CH= Hazen-Williams constant;R = Hydraulic mean depth, m; andS= Slope of energy line.

The continuity equation is given by:

$Q = AV$ (2) Where,Q = Flow of water, m^3/s; andA = Cross sectional area of flow, m^2.

Combining Equations (1) and (2), Equation (1) can be written in terms of Q as:

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$$h_f = \frac{10.674 L Q^{1.852}}{C_H^{1.852} D^{4.87}} \quad (3)$$

Where, hf = Loss of head due to friction, m; and L = Length of pipe, m.

Equation (3) can be rewritten as:

$$D = \left(\frac{10.674 L Q^{1.852}}{C_H^{1.852} h_f}\right)^{\frac{1}{4.87}} \quad (4)$$

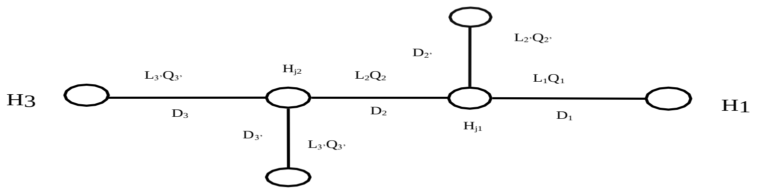
The head loss in a pipe can be calculated using Equation (3) for known values of C_H, Q and D. The value of C_H for CI pipe is 100 [7]. The required diameter of a pipe can be obtained from Equation (4) for known values of hf, C_H and Q. Thus, design of a pipe using Hazen-William equation is very simple because the value of C_H is constant for all types of flows.

IV. Description of Water Transmission System

A two junction water transmission system consists of the five pipes, four reservoirs and twojunctions as shown in Figure 1.For the purpose of optimal design of two junction water transmission system using the proposed model, the pipes, reservoirs and reservoir elevations have been assigned numbers using the following rules:

1. The pipes on the main line are assigned number 1 to 3 starting from the end of the system;
2. The last (the lowest reservoir) is assigned number 1 and the highest reservoir is assigned number 3;
3. The intermediate reservoirs are assigned number 2'and 3'; and
4. The junction on the mainline are assigned number j1,and j2.

H2'



H3'

Figure 1. Two junction water transmission system

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V. COST OF PIPES

Optimal design of a water transmission pipeline requires cost of its components. The cost of pipe is given by [8]:

$$C_m = k_m L_i D^m \qquad (5)$$ $_i$

Where,km and m = Pipe cost parameters; andDi= Diameter of the ith pipe, m. To determine cost of a pipe, the cost of CI pipe per metre length was taken from the Schedule of Rates[9]. The values of cost parameters for CI pipes were found to be km = 41072Rs. and m = 1.405 with $R^2 = 0.994$.

VI. System Development

To arrive at the objective function to be minimized, it is necessary to describe the cost function for the components of system. The design should be least costly as well as functionally effective.The objective function for a water transmission line consists of the cost of pipes and its accessories.The Lagrange multiplier technique appends the equality constraints to the objective function (cost function) through Lagrange multiplier to obtain the Lagrangian function. The minimization of Lagrangian function amounts to the minimization of the objective function as the constraint is satisfied. The optimal solution is achieved by taking the first partial derivative of the Lagrangian function with respect to the decision variables (diameter) and the Lagrange multiplier, and equating the expressions thus obtained to zero.

Two junction water transmission systems can be designed in two stages in which the first stage consists of three flow paths, and the second stage consists of two flowpaths. The flow paths 3-2-1 and 3-2-2’ contain 3 pipes as shown in Figs. 2 and 3.The two junction water transmission system consists of flow paths containing either two pipelines or three pipelines.For flow path 3-2-1, the total cost of the pipe line is given by:

$$C = Km\ L3D3^{m} + Km\ L2D2^{m} + Km\ L1\ D1^{m} \qquad (6)$$

H2’

L3Q3L2Q2

1 1

L Q

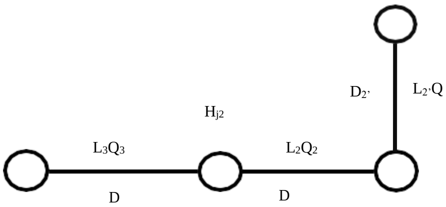
2’

H3

H1

D3

Hj2 D2



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Hj1 D1H3 Hj1

3 2

Figure2.Flow path 3-2-1 Figure3. Flow path 3-2-2’

The total head loss for flow path 3-2-1 is H3-H1. Using Equation (3), the total head loss constraint for flow path 3- 2-1 is given by:

$10.674 L3Q3^{1.852}$

$C^{1.852} D3^{1.852} + \underline{10.674 L2Q^{1.852}}$

2

$C^{1.852} D^{1.852} + \underline{10.674 L1Q^{1.852}}$

1

$C^{1.852} D^{1.852} = H3 - H1$(7)

H H 2 H 1

Combing Equations (6) and (7), the Lagrange function is given by:

$F = K_m L_3 D_3{}^m + K_m L_2 D_2{}^m + K_m L_1 D_1{}^m + \lambda\,(10.674 L3Q3^{1.852}$

$C^{1.852} D_3\ ^{1.852} + \underline{10.674 L2Q^{1.852}}$

2

$C^{1.852} D^{1.852} + \underline{10.674 L1Q^{1.852}}$

1

$C^{1.852} D^{1.852} - H_3 + H_1)$ (8)

H H 2 H 1

Differentiating Equation (8) with respect to D3,D2, D1 and λ, equating the results to zero, and after rearrangement and simplification, diameter equation for D3for flow path 3-2-1 (three pipes)can be written as:

1

4.870

1.852 4.870

1 1

1.852

4.870

F$\underline{10.674}$ ($1.852 + L\ \ Q^{1.852} Q3))^2 + 4.870 + L\ \ Q^{1.852} Q3)^{m} + 4.870)1$

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$D3$ =1.852 3 3

H×{(

Q2

H −H×{(} 3 1

Q1

(9)

I I

[ ]

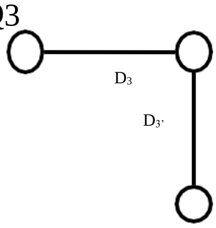
The diameter equation for D3 for flow path 3-2-2’ (three pipes) can be obtained from Equation(9) by replacing L1

by L2’,Q1 by Q2’, and H1 by H2’, respectively.

The flow paths 3-3’, 2-1 and 2-2’ contain 2 pipes as shown in Figures4, 5 and 6.

L3Q3

H3



Hj2

H2’

L3’Q3’L2'Q2'

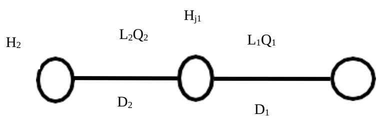
H1

H2

H3’

D2

Hj1



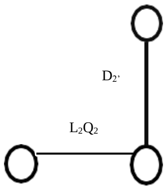


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Figure4. Flow path 3-3’ Figure5. Flow path 2-1 Figure6. Flow path 2-2’

Using similar procedure, the diameter equation for D3 for flow path 3-3’can be written as:

1

4.870

1.852

—

4.870

$\frac{10.674}{}$ (1.852+L Q1.852×{($\frac{Q3}{}$)m+4.870)1

C1.852 3 33′ 3′}

Q3′

D3= I$\frac{H}{}$ I

H3 −H3′(10)

I I

[ ]

Similarly, the diameter equation for D2 for flow path 2-1can be written as:

4.870

1.8521

—

4.870

$\frac{10.674}{}$ (L Q1.852+L Q1.852$\frac{Q2}{}$)}+4.870)1

1.852 2 2 c 1 1

H×{(

Q1

D2 =

H −H(11) 2 1

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I I[ ]

The diameter equation for D2 for flow path 2-2’ can be obtainedfrom Equation (11) by replacingL1 by L2’, Q1 by

Q2’and H1by H2’, respectively.

After calculating the diameter D3for the three flow pathsat stage one,and diameter D2 for two flow paths atstage two, select the maximum value of D3,andD2,and round offthese valuesto the next higher values at an increment of 50 mm.Using Equation (3), the head loss for pipe 3 is given by:

$$h_{f3} = \frac{10.674 L_3 Q_3^{1.852}}{C_H^{1.852} D_3^{4.87}} \quad (12)$$

Calculate the junction head Hj2, using reservoir head H3 and head loss hf3as follows:

Hj2 = H3- hf3 (13)

The head loss for pipe 3’ can be calculated using Equation (14) as follows:

hf3’ = Hj2- H3’ (14)

Using Equation (4), the diameter for pipe 3’ can be calculatedas follows:

$$D_{3'} = \left(\frac{10.674 L_{3'} Q_{3'}^{1.852}}{C_H^{1.852} h_{f3'}}\right)^{\frac{1}{4.87}} \quad (15)$$

Using Equation (3), the head loss for pipe 2 can be calculated.Using Equation (13), the junction headHj1can be calculated using Hj2 and hf2.Using Equation (4), the diameter for pipe 1 can be calculated.The value of hf2’can be calculated using Hj1 and hf2’.Using Equation (4), the diameter for pipe 2’ can be calculated.

Thediameters D3’, D1 and D2’are also rounded off to the next higher values at an increment of 50 mm. Thus, theoptimal design of two junction water transmission system can be obtained in two stages.

VII. Constraints Satisfaction

The basic goal is to minimize the cost of water transmission line subject to the relevant design criteria, hydraulic relationships, and constraints that may be linear as well as

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nonlinear. It is necessary to satisfy all the relevant constraints for each pipe as well as for each reservoir and junction of the water transmission system. The diameters obtained during the design process normally have fractional pipe sizes whereas the commercially available diameters have discrete values. Therefore, the diameter of pipes needs to be rounded off to the higher commercially available size.

VIII. Design Algorithm

A simple procedurefor the optimal design of two junction water transmission system has been developed that canbe used to determine optimal diameter, and optimal cost of two junction water transmission systems.The step by step design procedure for the optimal design of two junction water transmission systemsis as follow:

1. Determine diameter D3 for flow paths 3-2-1, 3-2-2'and 3-3' using Eqs. (9) and (10). The maximum value out ofthe three values of D3 is selected and rounded off to the next higher available size.
2. Calculate head loss for pipe 3 using Equation (12), and thereby calculate junction head Hj2.
3. Calculate head loss for pipe 3' and thereby calculatediameter D3'using Eq. (15).
4. Determine diametersD2 for flow path pipes2-1 and 2-2' using Eq. (11). The maximum value out of the two values of D2is selected and rounded off to the next higher available size.
5. Calculate head loss for pipe 2 using Eq. (12), and thereby calculate junction head Hj1.
6. Calculate head loss for pipes 2' and 1, and thereby calculate diameter D2' and D1 using Eq. (15).

The total optimal cost of two junction water transmission system consisting of cost pipes 3, 3', 2, 2' and 1 can be obtained using Equation (5) for the cost of pipe.

IX. Illustrative Design Example

The process of optimal design of two junction water transmission system is illustrated using an illustrative design example. The design data containing elevationof reservoirs, length of pipes and flow in each pipe are given in Table 1.

Table1:Design data for two junction water transmission system

Pipe / Reservoir	H, m	L, m	Q, m^3/s
3	100	5000	0.80
3'	70	2000	0.25
2	---	3000	0.55
2'	50	1000	0.15
1	30	4000	0.40

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X. Results and Discussions

Calculate diameter D3 at stage one for flow paths 3-2-1, 3-2-2’, and 3-3’ using Equations (9) and (10), and select the optimum value. Calculate diameter D3’ using Equation (15). Calculate diameter D2 at stage two for flow paths 2-1, and 2- 2’ using Equation (11), and select its optimum value. Calculate diameters D1and D2’using Eq. (15) as given in Table 2.

Table 2: Design details using flow path algorithm

Stage	Diameter	Flow path	Equation type	Calculated diameter, m	Selected diameter, m	Rounded diameter, m	Cost, Rs. Lakhs
1	D3	3-2-1	9	0.7263	0.776	0.800	1500.9
		3-2-2’	9	0.7363			
		3-3’	10	0.7756			
	D3’	---	15	0.5507	---	0.600	400.8
2	D2	2-1	11	0.6216	0.622	0.650	672.7
		2-2’	11	0.6185			
	D2’	---	15	0.4115	---	0.450	133.8
	D1	---	15	0.5659	---	0.600	801.5
Total							3509.7

To validate the results obtained by the flow path algorithm, the optimal design was also obtained using exhaustive search method. Fourfeasible options for the diameterD3,and the corresponding feasible values of diameterD2were considered as given in Table 3. It is obvious from Table 3 that there is one value of diameter D2 that yields minimum cost solution for each value of diameter D3.

Table 3: Diameters and corresponding cost using exhaustive search method

Diameter, m					Cost, Rs. Lakhs
D3	D3’	D2	D2’	D1	
0.750	0.750(0.715)	0.650 (0.642)	0.450 (0.414)	0.600 (0.567)	3527.1
		0.700	0.400 (0.355)	0.550 (0.545)	3488.3
		0.750	0.350 (0.334)	0.550 (0.533)	3544.9
0.776	0.551	0.622 (0.6216)	0.412 (0.4115)	0.566 (0.5659)	3282.5
0.800 (0.776)	0.550 (0.502)	0.600	0.450 (0.424)	0.600 (0.569)	3392.0
		0.650 (0.613)	0.350 (0.339)	0.550 (0.536)	3331.5
		0.700	0.350 (0.316)	0.550 (0.520)	3405.3
		0.750	0.350 (0.304)	0.550 (0.511)	3481.3
0.850	0.500	0.600 (0.599)	0.400 (0.358)	0.550 (0.546)	3368.3
		0.650	0.350 (0.315)	0.550 (0.520)	3420.5

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	(0.458)	0.700	0.300 (0.298)	0.550 (0.506)	3476.0
0.900	0.500 (0.437)	0.600 (0.592)	0.350 (0.334)	0.550 (0.533)	3485.6
		0.650	0.350 (0.303)	0.550 (0.510)	3557.2
		0.700	0.300 (0.289)	0.500 (0.497)	3523.8

The variation in the total cost of system with diameter D2for each value of diameter D3is shown in Fig. 7. Further, the variation in the minimum cost of system with diameter D3 is shown in Fig. 8.

Thus, the optimal solution obtained by considering 2 to 4 feasible options for D2 yields the optimal solution with D3 = 0.800 m (Fig. 8). The optimal design corresponds to D3 = 0.800 m, and D2 = 0.650 m with optimal cost as 3331.5 Rs. Lakhs (Table 3). Thus, the solution obtained by the flow path algorithm and the exhaustive search method is the same. The optimal diameters for pipes 3, 3', 2, 2', 1 are 0.800 m, 0.550 m, 0.650 m, 0.350 m, and 0.550 m, respectively (Table 3).

3600

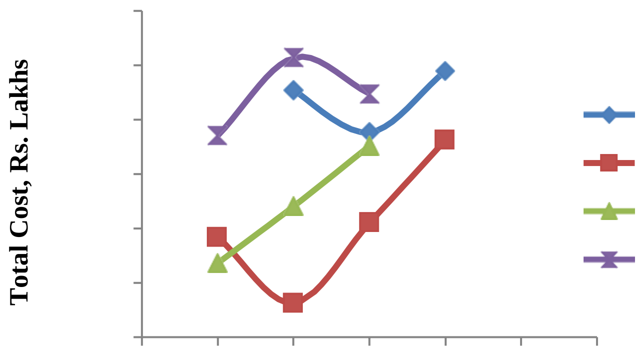
3550

3500

3450

3400

3350



D3= 0.750

D3= 0.800

D3= 0.850

D3= 0.900

3300

0.55 0.6 0.65 0.7 0.75 0.8 0.85

Diameter D_2, m

Figure 7.Variation in cost of system with diameter of pipe 2

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3500

3480

3460

3440

3420

3400

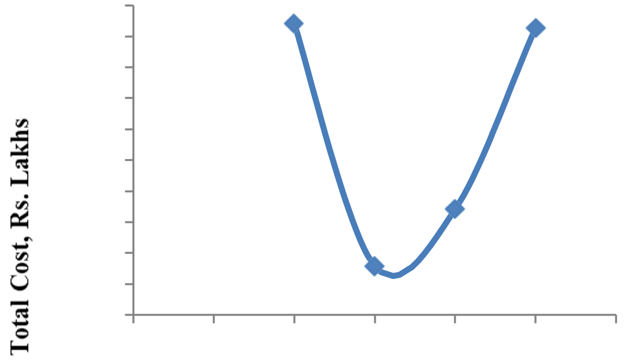
3380

3360

3340

3320

3300



0.65 0.7 0.75 0.8 0.85 0.9 0.95

Diameter D_3, m

Figure8.Graph of the minimum cost option versus diameter of pipe 3

XI. Conclusions

Following conclusions can be drawn from this study:

[1] Flow path algorithm can be used for the optimal design of two junctionwater transmission system;

[2] The cost function in power form can be used to obtain the cost of pipe;

[3] The method converts the complex system into simple system by breaking down the multi junction watertransmission networks in flow paths;

[4] The flow path algorithm is very easy and yields the optimal design in less time.

[5] The solution obtained by flow path algorithm is optimal;

Further, the flow path algorithm can be extended, with modification, to obtain the optimal design of multi reservoirmulti junction water transmission system and the result can be obtained in less time.

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