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Some More Properties of Micro SP-Continuous using Micro SP-open Sets.

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Abstract:
The objective of this paper is to study some more properties of Micro SP-continuous in Micro SP-topological spaces. Also we define and analyze Micro SP-homeomorphism in terms of Strongly Micro-continuous and Strongly Micro SP-open map.

Keywords:
Micro SP-open, Micro SP-closed, Micro-continuous, Micro SP-continuous, Strongly Micro SP-continuous, Strongly Micro SP-open map, Micro SP-homeomorphism.

1. Introduction

Continuous function plays a vital role in Topological spaces. In 2019, Continuous function of Micro topological spaces was proposed by Chandrasekar.S [1]. Taha H.Jasimet al [8] generalized the Micro-continuous function to the concept of Micro generalized irresolute function in 2021. Hariwan Z Ibrahim [4] defined the mapping which is Micro-closed map in 2020. In 2022, Micro SP-continuous function in Micro topological spaces is introduced by Hardi A.Shareef [3]. In this paper some more properties of Micro SP-continuous functions are studied. Also the new class of functions called Strongly Micro SP-continuous function and Strongly Micro SP-open map are defined and few results involving their equivalent characterizations are derived. In addition, Micro SP-homeomorphism is defined as well as certain remarkable properties are deduced.

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2. Preliminaries

Definition2.1.[1] Let(U,τR(X)) be a Nano topological space. Then (U,τR(X))={N $\cup$(N′$\cap\mu$):N,N′$\in\tau$R(X)} and $\mu\notin\tau$R(X) is called the Micro topology on U with respect to The triplet (U,τR(X)), μR(X)) is called Micro topological space and the elements of μR(X) are Micro-open sets and the complement of Micro-open set is called a Micro-closed set.

Definition 2.2.[5] Let (U, τR(X), μR(X)) be a Micro topological space and A $\subseteq$ U. Then A is said to be Micro SP-open (briefly Mic SP-open) if for each $x\in$ A $\in$ Mic-SO(U,X) there exists a micro pre-closed set F such that $x\in$ F $\subseteq$ A. The collection of all Micro SP-open sets is denoted by Mic SP-O (U,X).

Definition 2.3. [5] Let(U, τR(X), μR(X)) be a Micro topological space. A subset B of U is called Micro SP-closed (briefly Mic SP-closed) if and only if its complement is Micro SP-open and Mic SP-CL (U,X)denotes the set of all Micro SP-closed sets.

Definition 2.4. [5] For any two subsets A and B of a Micro topological space (U,τR(X), μR(X)), the following properties are true.

(i) Mic SP-Int(Mic SP-Int(A)) – MicSP-Int(A).
(ii) Mic SP-Int(A) = U – Mic SP-Cl(U – A)
(iii) If A $\subseteq$ B, then Mic SP-Cl(A) $\subseteq$ Mic SP-Cl(B)
(iv) Mic SP-Cl(A) $\cup$ Mic SP-Cl(B) $\subseteq$ Mic SP-Cl(A $\cup$ B)
(v) Mic SP-Cl (A $\cap$ B) $\subseteq$ Mic SP-Cl(A) $\cap$ Mic SP-Cl(B).

Definition 2.5. [5] For any two subsets A and B of a Micro topological space (U,τR(X), μR(X)), the following properties are true.

(i) Mic SP-Cl(Mic SP-Cl(A)) = MicSP-Cl(A).
(ii) Mic SP-Cl(A) = U – Mic SP-Int(U – A)
(iii) IfA$\subseteq$B,thenMicSP-Cl(A)$\subseteq$MicSP-Cl(B)
(iv) Mic SP-Cl(A) $\cup$ Mic SP-Cl(B) $\subseteq$ Mic SP-Cl(A $\cup$ B)
(v) MicSP-Cl(A $\cap$ B) $\subseteq$ Mic SP-Cl(A) $\cap$ Mic SP-Cl(B).

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Definition 2.6. [1] Let (U, τR(X), μR(X)) and V, τR(Y), μR(Y)) be two Micro topological spaces. A function f : U → V is said to be Micro-continuous function if f^{-1} (A) is Micro-open in U for every Micro-open set A in V.

Definition 2.7 [2] Let U, τR(X), μR(X)) and V, τR(Y), μR(Y)) be two Micro topological spaces. A function f: U → V is said to be Micro-closed map if for any Micro closed set B in V, f(A) is Micro-closed in U.

Definition 2.8 [3] A function f:V→V′ where (V,τ⌈(X),μ⌈(X) and where (V′,τ′⌈′(Y), μ′⌈′(Y) are Micro topological spaces is called Micro SP-continuous function at a point x ∈V iff or each Micro open set H in V′ containing f(x). There exists a Micro SP-open set K in V containing x such that f (K) ⊆ H.

3. Properties of Micro SP-continuous functions

Theorem3.1. Afunction$f$:U→V is Micro SP-continuous if and only if each of the following holds.

(i) The inverse image of every Micro-closed set in V is Micro SP-closed in U.

(ii) Mic SP-Cl[f^{-1}(F)⊆f^{-1}[Mic-Cl(F)] for all F ⊆ V.

(iii) f^{-1} [Mic-Int(H) ⊆ Mic SP-Int[f^{-1}(B)] for all H ⊆ V.

Proof:

(i) Necessity: Let f be Micro SP-continuous and F ∈ Mic Cl(V, Y)That is (V – F) ∈ Mic-O(V,Y).Since f is Micro SP-continuous, f^{-1}(V – F)∈ Mic SP-(U, X).That is (U – f^{-1}(F) ∈ Mic SP-O(U,X) implies that f^{-1}(F)∈ Mic SP-CL(U,X).Thus the inverse image of every Micro-closed set in V is Micro SP-closed in U, if f is Micro SP-continuous on U.

Micro SP-closed in U, iff is Micro SP-continuous on U.

(ii) Necessity: If f is Micro SP-continuous and F ⊆ V and Mic-Cl(F) ∈ Mic-CL(V,Y) and from (i), f^{-1} (Mic-Cl(F)) SP-CL(U,X) implies Mic SP-Cl(f^{-1} (Mic-Cl(F))) = f^{-1} (Mic-Cl(F)). Since F ⊆ Mic-Cl(F) which implies f^{-1}(F)⊆f^{-1}(Mic-Cl(F)). Thus Mic SP-Cl(f^{-1} (F)) ⊆ Mic-Cl(f^{-1} (Mic-Cl(F))) = f^{-1}(Mic-Cl(F)).That is Mic SP-Cl(f^{-1}(F)) ⊆ f^{-1} (Mic-Cl(F)).

Sufficiency: Let Mic SP-Cl(f^{-1} (F)) ⊆ f^{-1}(Mic-Cl(F)) for every F ⊆ V.Let F ∈ Mic-CL(V,Y). Then Mic-Cl(F) = F. By assumption, Mic SP-Cl(f^{-1}(F)) ∈ f^{-1}(Mic-Cl(F)) = f^{-1}(F). Thus Mic SP-

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Cl(f^{-1} (F)) $\subseteq$ f^{-1}(F). But f^{-1} (F) $\subseteq$ Mic SP-Cl(f^{-1} (F)). There Mic SP-Cl(f^{-1} (F)) =f^{-1}(F).Thusf^{-1}(F) is Micro SP-closed in U for every Micro-closed set F in V. Hence f is Micro SP-continuous.

(iii) Necessity: Let f be Micro SP-continuous and B $\subseteq$ V. Then Mic-Int(B) $\in$ Mic-O(V,Y) Therefore f^{-1} (Mic-Int(B)) $\in$ Mic SP-O(U, X). That is f^{-1}(Mic-Int(B)) = Mic SP-Int(f^{-1}(Mic-Int(B))). Also Mic-Int(B) $\subseteq$ B implies Mic SP-Int(f^{-1}(Mic-Int(B))) $\subseteq$ Mic-Int(f^{-1}(B)). Therefore

f^{-1}(Mic-Int(B))=MicSP-Int(f^{-1}(Mic-Int(B))) $\subseteq$ Mic SP-Int(f^{-1}(B)). That is f^{-1}(B) = f^{-1}(Mic-Int(B) $\subseteq$ Mic SP-Int(f^{-1}(B)).

Sufficiency: Let f^{-1} (Mic-Int(B)) $\subseteq$ Mic SP-Int(f^{-1}(B)) for every subset B of V. If B $\in$ Mic-O(V,Y), F = Mic-Int(B). Also f^{-1}(B) = f^{-1}(Mic-Int(B)), but f^{-1}(Mic-Int(B)), but = f^{-1}(Mic-Int(B)) $\subseteq$ Mic SP-Int(f^{-1}(B)). That is f^{-1}(B) = f^{-1}(Mic-Int(B)) $\subseteq$ Mic SP-Int(f^{-1}(B)). Thus f^{-1}(B) = MicSP-Int(f^{-1}(B)) which implies f^{-1}(B) is Micro SP-open in U for every Micro-open set B in V. Hence f is Micro SP-continuous.

Definition 3.2. Let (U, τR(X), μR(X)) and (V, τR(Y), μR(Y)) be two Micro topological spaces. A function f : U $\to$V is said to be Strongly Micro SP-continuous function if f^{-1}(B) is Micro-open in U for every Micro SP-open set B in V.

Example 3.3. Let U={a,b,c,d},with U|R = {{a,d},{b},{c}}and X = {c,d}}, Then τR(X) = {U, ϕ,{c},{a,d},{a,c,d}}. If μ = {b} then μR(X) = {U,ϕ,{b},{a,d},{b,c},{a,b,d},{a,c,d}}. Also V=
{1,2,3,4} with V|R = {{1},{2,3},{4}} and Y = {2,4}.Then τR(Y) = (V,ϕ, {4},{2,3},{2,3,4}}. If μ = {1} then μR(Y) = {V,ϕ, {1},{4},{1,4},{2,3},{1,2,3},{2,3,4} and Micro SP-open sets in V are {V,ϕ, {1},{4},{1,4},{2,3},{1,2,3},{2,3,4}}.

Define f: U $\to$ V as f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 3. Here f^{-1}({1}) = {b}, f^{-1}({4}) = {c}, f^{-1}({1,4}) = {b,c}, f^{-1}({2,3}) = {a,d}, f^{-1}({1,2,3}) = {a,b,d}, f^{-1}({2,3,4}) = {a,c,d}, f^{-1}(U) = V.

Hence f is Strongly Micro SP-continuous.

Theorem 3.4. A function f: U$\to$V is Strongly Micro SP-continuous if and only if each of the following holds.

(i) The inverse image of every Micro SP-closed set in V is Micro-closed in U.

(ii) The inverse image of every Micro SP-open set in V is Micro-open in V.

(iii) Mic-Cl[f^{-1}(F)] $\subseteq$ f^{-1}[Mic SP-Cl(F)] for all F $\subseteq$ V.

(iv) f^{-1}[Mic SP-Int(B)]=Mic-Int[f^{-1}(B)] for all B $\subseteq$ V.

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Proof: (i) Necessity: Let f be Strongly Micro SP-continuous and F $\in$Mic SP-Cl(V,Y). That is, (V – F) $\in$ Mic SP-O(V,Y). Since f is Strongly Micro SP-continuous. f^{-1}(V – F) $\in$ Mic-O(U,X) and (U–f^{-1}(F)) $\in$ Mic-O(U,X). Hence f^{-1}(F) $\in$ Mic-CL(U,X).

Sufficiency: Let f^{-1} (F) $\in$ Mic-CL(U,X), for all F $\in$ Mic SP-CL(V,Y). Let B $\in$ Mic SP-O(V,Y). Then (V – B) $\in$ Mic SP-CL(V,Y). Then f^{-1} (V – B) $\in$ Mic-CL(U,X), that is (U – f^{-1} (F) $\in$ Mic-CL(U,X) implies f^{-1} (F) $\in$ Mic-O(U,X). Hence the inverse image of every Micro SP-open set in V is Micro-open in U. Therefore f is strongly Micro SP-continuous on U.

(ii) Necessity: Let f be strongly Micro SP-continuous and B be any Micro SP-open set in V. If f^{-1} (B) = ϕ, then f^{-1} (B) is Micro-open in U.Iff^{-1} (B) $\neq \phi$, then there exists a Micro-open set A in U containing x such that f(A) $\subseteq$ B which implies x $\in$ A $\subseteq f^{-1}$ (B) and hence f^{-1} (B) is Micro S**P**-open.

Sufficiency: Let B be any Micro-open set in V containing f^{-1} (x), then x $\in f^{-1}$ (B) and by hypothesis f^{-1} (B) is a Micro-open set in U containing x, so f (f^{-1} (B)) $\subseteq$ B. Hence f is Strongly Micro SP-continuous.

(iii) Necessity: If f is Strongly Micro SP-continuous and F $\subseteq$ V, Mic SP-Cl(F) $\in$ Mic SP-CL(V,Y) and from (i) f^{-1} (Mic SP-Cl(F))) = f^{-1} (Mic SP-Cl(F))) $\in$ Mic-CL(U,X) and Mic-Cl(f^{-1}(Mic SP-Cl(F))) = f^{-1}(Mic SP-Cl(F))). Since F $\subseteq$ Mic SP-Cl(F) which implies f^{-1}(F) $\subseteq$ f^{-1}(Mic SP-Cl(F))) implies that Mic-Cl(f^{-1}(F)) $\subseteq$ Mic-Cl(f^{-1}(MicSP-Cl(F))) = f^{-1}(Mic SP-Cl(F))). Hence Mic-Cl(f^{-1}(F)) $\subseteq$ (f^{-1}(Mic SP-Cl(F))).

Sufficiency: Let Mic-Cl(f^{-1} (F)) $\subseteq f^{-1}$ (Mic SP-Cl(F)), for every F $\subseteq$ V. Let F $\in$ Mic SP-CL(V,Y). Then Mic SP-Cl(F) = F. By assumption, Mic-Cl(f^{-1} (F)) $\subseteq f^{-1}$(MicSP-Cl(F)) = f^{-1}(F). Thus Mic-Cl(f^{-1}(F). But f^{-1}(F) $\subseteq$ Mic-Cl(f^{-1} (F)). Therefore Mic-Cl(f^{-1} (F)) = f^{-1}(F)) where f^{-1} (F) $\in$ Mic-Cl(U,X), for every Micro SP-closed set F in V. Hence f is Strongly Micro SP-continuous on U.

(iv) Necessity: Let f be strongly Micro SP-continuous and B $\subseteq$ V. Then Mic SP-Int(B)$\in$ Mic SP-O(V,Y). Thus f^{-1} (Mic SP-Int(B))) $\in$ Mic-O(U,X). That is f^{-1} (Mic SP-Int(B))=

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Mic-Int(f^{-1}Mic SP-Int(B))). Also Mic SP-Int(B) $\subseteq$ B implies that Mic-Int(f^{-1} (Mic SP-Int(B))) $\subseteq$ Mic-Int(f^{-1}(B)). Thus f^{-1} (MicSP-Int(B)) = Mic-Int(f^{-1}(Mic SP-Int(B))) $\subseteq$ Mic-Int(f^{-1} (B)). Hence f^{-1}(Mic SP-Int(B)) $\subseteq$ Mic-Int(f^{-1} (B)).

Sufficiency: Let (f^{-1}Mic SP-Int(B)) $\subseteq$ Mic-Int((f^{-1}(B)), for every subset B of V. If B is Micro SP-open in V, B = Mic SP-Int(B). Also f^{-1}(B) = f^{-1} (Mic SP-Int(B)), but f^{-1} (Mic SP-Int(B)) $\subseteq$ Mic-Int((f^{-1}(B)) implies that f^{-1}(B) =f^{-1} (Mic SP-Int(B)) $\subseteq$ Mic-Int((f^{-1}(B)). Therefore f^{-1}(B) = Mic-Intf^{-1}(B)). Thus f^{-1}(B) is Micro-open in U for every Micro SP-open set B inV. Hence f is Strongly Micro SP-continuous.

Definition 3.5. Let ((U, τR(X), μR(X)), (V, τR(Y), μR(Y)) be two Micro topological spaces. Then a mapping. f: U $\rightarrow$ V is strongly Micro SP-open map if the image of every Micro SP-open set in U is Micro-open in V. The mapping f is said to be Strongly Micro SP-closed map if the image of every Micro SP-closed set in U is Micro-closed set in V.

Example 3.6. Let U = {a,b,c,d} with U|R = {{a,d},{b},{c}} and X = {c,d}, Then τR(X) = {U,ϕ,
{c},{a,d},{a,c,d}}.{a,c,d}}. If μ = {b} then μR(X) = {U,ϕ,{b},{c},{a,d},{b,c},{a,b,d}, {a,c,d}}. Here{U,ϕ,{b},{c},{b,c},{a,b,d} are Micro SP-open sets in U. Also let V = {1,2,3,4} with V|R = {{1},{3},{2,4}} and Y = {2,4}. ThenτR(Y) = {V,ϕ,{2,4}}. If μ = {1} then μR (Y) =
{V,ϕ,{1},{2,4},{1,2,4}} which are Micro-open sets in V.

Define f: U$\rightarrow$V as f(a) = 2, f(b) = 1, f(c) = 1, f(d) = 4.Then the image of every Micro SP-open set in U is Micro-open in V. Hence f is Strongly Micro SP-open map.

4. Micro SP-homeomorphism

Definition 4.1. Let ((U, τR(X),μR(X)), (V, τR(Y)),μR(Y)) be two Micro topological spaces. A bijective function f: U$\rightarrow$ V is said to be Micro SP-homeomorphism if f and $f^{-1}$are both Micro SP-continuous.

Example 4.2. Let U = {a,b,c,d) with U|R = {{a},{b,c},{d}} and X = {b,d}. Then τR(X) = {U,ϕ,{c}, {d},{b,c},{b,c,d}}. If μ={a} then μR(X) = {U, ϕ, {a}, {d}, {a,d},{b,c},{a,b,c},{b,c,d}}. Micro SP-open sets in U are {U,ϕ,{a},{d},{a,d},{b,c},{a,b,c}, {b,c,d}}. Also let V = {1,2,3,4} with V|R = {{1,4},{2},{3}} and Y = {3,4}. Then τR(Y) = {V, ϕ,{3},{1,4},{1,3,4}}. If μ = {2} then τR (Y) =
{V,ϕ,{2},{3},{1,4},{2,3},{1,2,4},{1,3,4}}.

Define a bijective function f: U $\rightarrow$ V as f (a) = 2, f (b) =1, f (c)=4, f(d}=3.Here f^{-1}({2}) = {a}, f^{-1}({3})={d}, f^{-1}({1,4}) = {b,c}, f^{-1}({2,3}) = {a,d}, f^{-1}({1,2,4}) = {a,b,c}, f^{-1}({1,3,4}) = {b,c,d} and f^{-1}(V) = U. Therefore f is Micro SP-continuous. Also, here f({a}) = {2}, f({d}) = {3}, f({b,c}) = {1,4},f({a,d}) = {2,3}, f({a,b,c}) = {1,2,4}, f({b,c,d}) = {1,3,4} and f(U) = V. Thus
$f^{-1}$is Micro SP-continuous. Here both f and $f^{-1}$are Micro SP-continuous. Hence f is Micro SP-Homeomorphism.

Definition 4.3 Let (U, τR(X),μR(X)), (V, τR(Y)),μR(Y)) be two Micro topological spaces. Then a bijective function f: U $\rightarrow$ V is said to be Strongly Micro SP-homeomorphism if f and f^{-1} are both Strongly Micro SP-continuous.

Example 4.4 Let U = {a,b,c,d}, with U|R = {{a,d},{b},{c}} and X = {c,d}. Then τR(X) = {U,ϕ,
{c}, {a,d},{a,c,d}}. If μ = {b} then μR(X) = {U, ϕ,{b},{c},{a,d},{b,c},{a,b,d},{a,c,d}}. Micro SP-open sets in U are {U,ϕ,{b},{c},{b,c},{a,b,d}}. Also let V = {1,2,3,4} with V|R = {{1},
{2,3}, {4}} and Y = {2,4}. Then τR (Y) = {V,ϕ,{4},{2,3},{2,3,4}. If μ = {1} then μR (Y) = {V, ϕ,{1},{4},{1,4},{2,3},{1,2,3},{2,3,4}} which are Micro-open sets in V.

Define a bijective function f: U $\rightarrow$ V as f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 3. Here f^{-1}({1}) = {b},
f^{-1}({4}) = {c}, f^{-1} ({1,4}) = {b,c}, f^{-1} ({2,3}) = {a,d}, f^{-1} ({1,2,3}) = {a,b,d}, f^{-1} ({2,3,4}) = {a,c,d} and f^{-1}(U) =V. Thus f is Strongly Micro SP-continuous. Also, here f({b}) = {1}, f ({c}) = {4}, f ({b,c}) = {1,4}, f ({a,b,d}) = {1,2,3} and f (U) = V. Therefore $f^{-1}$is Strongly Micro SP-continuous. Here both f and $f^{-1}$are Strongly Micro SP-continuous. Hence f is Strongly Micro SP-homeomorphism.

5. Conclusion

In this paper, we studied some more properties of Micro SP-continuous functions and expounded their relations with Strongly Micro SP-continuous and Strongly Micro SP-open map. Also we launched a Micro SP-homeomorphism concepts. Furthermore, this work will be extended to establish the new concepts of Micro SP-irresolute, Micro SP-open map and Micro SP-closed map and their relative properties will be derived. In future, Micro SP-homeomorphism concepts can be applied in real life situations through some Micro topological structures.

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