Some More Properties of Micro SP-Continuous using Micro SP-open Sets.

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Abstract:
The objective of this paper is to study some more properties of Micro SP-continuous in Micro SP-topological spaces. Also we define and analyze Micro SP-homeomorphism in terms of Strongly Micro-continuous and Strongly Micro SP-open map.

Keywords:

1. Introduction
Continuous function plays a vital role in Topological spaces. In 2019, Continuous function of Micro topological spaces was proposed by Chandrasekar.S [1]. Taha H.Jasim et al [8] generalized the Micro-continuous function to the concept of Micro generalized irresolute function in 2021. Hariwan Z Ibrahim [4] defined the mapping which is Micro-closed map in 2020. In 2022, Micro SP-continuous function in Micro topological spaces is introduced by Hardi A.Shareef [3]. In this paper some more properties of Micro SP-continuous functions are studied. Also the new class of functions called Strongly Micro SP-continuous function and Strongly Micro SP-open map are defined and few results involving their equivalent characterizations are derived. In addition, Micro SP-homeomorphism is defined as well as certain remarkable properties are deduced.
2. Preliminaries

**Definition 2.1.**[1] Let \((U, \tau R(X))\) be a Nano topological space. Then \((U, \tau R(X))=\{N \cup (N' \cap \mu) : N, N' \in \tau R(X)\}\) and \(\mu \in \tau R(X)\) is called the Micro topology on \(U\) with respect to The triplet \((U, \tau R(X), \mu R(X))\) is called Micro topological space and the elements of \(\mu R(X)\) are Micro-open sets and the complement of Micro-open set is called a Micro-closed set.

**Definition 2.2.**[5] Let \((U, \tau R(X), \mu R(X))\) be a Micro topological space and \(A \subseteq U\). Then \(A\) is said to be Micro S\(\text{P}\)-open (briefly Mic S\(\text{P}\)-open) if for each \(x \in A\) there exists a micro pre-closed set \(F\) such that \(x \in F \subseteq A\). The collection of all Micro S\(\text{P}\)-open sets is denoted by Mic S\(\text{P}\)-O \((U, X)\).

**Definition 2.3.**[5] Let \((U, \tau R(X), \mu R(X))\) be a Micro topological space. A subset \(B\) of \(U\) is called Micro S\(\text{P}\)-closed (briefly Mic S\(\text{P}\)-closed) if and only if its complement is Micro S\(\text{P}\)-open and Mic S\(\text{P}\)-CL \((U, X)\) denotes the set of all Micro S\(\text{P}\)-closed sets.

**Definition 2.4.**[5] For any two subsets \(A\) and \(B\) of a Micro topological space \((U, \tau R(X), \mu R(X))\), the following properties are true.

(i) Mic S\(\text{P}\)-Int(Mic S\(\text{P}\)-Int(A)) = MicSP-Int(A).
(ii) Mic S\(\text{P}\)-Int(A) = U – Mic SP-Cl(U – A)
(iii) If \(A \subseteq B\), then Mic SP-Cl(A) \(\subseteq\) Mic SP-Cl(B)
(iv) Mic SP-Cl(A) \(\cup\) Mic SP-Cl(B) \(\subseteq\) Mic SP-Cl(A \(\cup\) B)
(v) Mic SP-Cl (A \(\cap\) B) \(\subseteq\) Mic SP-Cl(A) \(\cap\) Mic SP-Cl(B).

**Definition 2.5.**[5] For any two subsets \(A\) and \(B\) of a Micro topological space \((U, \tau R(X), \mu R(X))\), the following properties are true.

(i) Mic SP-Cl(Mic SP-Cl(A)) = MicSP-Cl(A).
(ii) Mic SP-Cl(A) = U – Mic Sp-Int(U – A)
(iii) If \(A \subseteq B\), then Mic SP-Cl(A) \(\subseteq\) Mic SP-Cl(B)
(iv) Mic SP-Cl(A) \(\cup\) Mic SP-Cl(B) \(\subseteq\) Mic SP-Cl(A \(\cup\) B)
(v) MicSP-Cl(A \(\cap\) B) \(\subseteq\) Mic SP-Cl(A) \(\cap\) Mic SP-Cl(B).
**Definition 2.6.** [1] Let \((U, \tau(U), \mu(U))\) and \(V, \tau(V), \mu(V)\) be two Micro topological spaces. A function \(f: U \rightarrow V\) is said to be Micro-continuous function if \(f^{-1}(A)\) is Micro-open in \(U\) for every Micro-open set \(A\) in \(V\).

**Definition 2.7** [2] Let \((U, \tau(U), \mu(U))\) and \(V, \tau(V), \mu(V)\) be two Micro topological spaces. A function \(f: U \rightarrow V\) is said to be Micro-closed map if for any Micro closed set \(B\) in \(V, f(A)\) is Micro-closed in \(U\).

**Definition 2.8** [3] A function \(f: V \rightarrow V'\) where \((V, \tau'(V), \mu'(V))\) and \((V', \tau'(V), \mu'(V))\) are Micro topological spaces is called Micro SP-continuous function at a point \(x \in V\) iff or each Micro open set \(H\) in \(V'\) containing \(f(x)\) there exists a Micro SP-open set \(K\) in \(V\) containing \(x\) such that \(f(K) \subseteq H\).

### 3. Properties of Micro SP-continuous functions

**Theorem 3.1.** A function \(f: U \rightarrow V\) is Micro SP-continuous if and only if each of the following holds.

1. The inverse image of every Micro-closed set in \(V\) is Micro SP-closed in \(U\).
2. \(\text{Mic SP-Cl}(f^{-1}(F)) \subseteq f^{-1}([\text{Mic Cl}(F)])\) for all \(F \subseteq V\).
3. \(f^{-1}([\text{Mic Int}(H)]) \subseteq \text{Mic SP-Int}(f^{-1}(B))\) for all \(H \subseteq V\).

**Proof:**

1. **Necessity:** Let \(f\) be Micro SP-continuous and \(F \subseteq \text{Mic Cl}(V, Y)\). That is \((V - F) \subseteq \text{Mic O}(V,Y)\). Since \(f\) is Micro SP-continuous, \(f^{-1}(V - F) \subseteq \text{Mic Sp}(U, X)\). That is \((U - f^{-1}(F)) \subseteq \text{Mic SP O}(U,X)\) implies that \(f^{-1}(F) \subseteq \text{Mic SP CL}(U,X)\). Thus the inverse image of every Micro-closed set in \(V\) is Micro SP-closed in \(U\), if \(f\) is Micro SP-continuous on \(U\).

   **Micro SP-closed in \(U\), iff is Micro SP-continuous on \(U\).**

2. **Necessity:** If \(f\) is Micro SP-continuous and \(F \subseteq V\) and \(\text{Mic Cl}(F) \subseteq \text{Mic CL}(V,Y)\) and from (i), \(f^{-1}([\text{Mic Cl}(F)]) \subseteq \text{Mic Cl}(U,X)\) implies \(\text{Mic SP CL}(f^{-1}([\text{Mic Cl}(F)])) = f^{-1}([\text{Mic Cl}(F)])\). Since \(F \subseteq \text{Mic Cl}(F)\) which implies \(f^{-1}(F) \subseteq f^{-1}([\text{Mic Cl}(F)])\). Thus \(\text{Mic SP Cl}(f^{-1}(F)) \subseteq \text{Mic Cl}(f^{-1}([\text{Mic Cl}(F)])) = f^{-1}([\text{Mic Cl}(F)])\). That is \(\text{Mic SP CL}(f^{-1}(F)) \subseteq f^{-1}([\text{Mic Cl}(F)])\).

   **Sufficiency:** Let \(\text{Mic SP Cl}(f^{-1}(F)) \subseteq f^{-1}([\text{Mic Cl}(F)])\) for every \(F \subseteq V\). Let \(F \subseteq \text{Mic Cl}(V,Y)\). Then \(\text{Mic Cl}(F) = F\). By assumption, \(\text{Mic SP Cl}(f^{-1}(F)) \subseteq f^{-1}([\text{Mic Cl}(F)]) = f^{-1}(F)\). Thus Micro SP-
The inverse image of every Micro SP-closed set in V is Micro-closed in U.

(i) The inverse image of every Micro SP-open set in V is Micro-open in U.

(ii) The inverse image of every Micro SP-open set in V is Micro-open in V.

(iii) Mic-Cl\(f^{-1}(F)\) \(\subseteq f^{-1}[\text{Mic SP-Cl}(F)]\) for all \(F \subseteq V\).

(iv) \(f^{-1}[\text{Mic SP-Int}(B)]=\text{Mic-Int}[f^{-1}(B)]\) for all \(B \subseteq V\).

**Definition 3.2.** Let \((U, \tau R(X), \mu R(X))\) and \((V, \tau R(Y), \mu R(Y))\) be two Micro topological spaces. A function \(f: U \to V\) is said to be Strongly Micro SP-continuous function if \(f^{-1}(B)\) is Micro-open in U for every Micro SP-open set B in V.

**Example 3.3.** Let \(U=\{a,b,c,d\}\), with \(U|R=\{\{a,d\},\{b\},\{c\}\}\) and \(X=\{c,d\}\). Then \(\tau R(X)=\{U,\phi,\{c\},\{a,d\},\{a,c,d\}\}\). If \(\mu = \{b\}\) then \(\mu R(X)=\{U,\phi,\{b\},\{a,d\},\{b,c\},\{a,b,d\},\{a,c,d\}\}\). Also V= \(\{1,2,3,4\}\) with \(V|R=\{\{1\},\{2,3\},\{4\}\}\) and \(Y=\{2,4\}\). Then \(\tau R(Y)=\{V,\phi,\{4\},\{2,3\},\{2,3,4\}\}\). If \(\mu = \{1\}\) then \(\mu R(Y)=\{V,\phi,\{1\},\{4\},\{1,4\},\{2,3\},\{1,2,3\},\{2,3,4\}\}\) and Micro SP-open sets in V are \(\{V,\phi,\{1\},\{4\},\{1,4\},\{2,3\},\{1,2,3\},\{2,3,4\}\}\).

Define \(f: U \to V\) as \(f(a)=2, f(b)=1, f(c)=4, f(d)=3\). Here \(f^{-1}(\{1\})=\{b\}, f^{-1}(\{4\})=\{c\}, f^{-1}(\{1,4\})=\{b,c\}, f^{-1}(\{2,3\})=\{a,d\}, f^{-1}(\{1,2,3\})=\{a,b,d\}, f^{-1}(\{2,3,4\})=\{a,c,d\}, f^{-1}(U)=V\).

Hence \(f\) is Strongly Micro SP-continuous.

**Theorem 3.4.** A function \(f: U \to V\) is Strongly Micro SP-continuous if and only if each of the following holds.

(i) The inverse image of every Micro SP-closed set in V is Micro-closed in U.

(ii) The inverse image of every Micro SP-open set in V is Micro-open in U.

(iii) Mic-Cl\(f^{-1}(F)\) \(\subseteq f^{-1}[\text{Mic SP-Cl}(F)]\) for all \(F \subseteq V\).

(iv) \(f^{-1}[\text{Mic SP-Int}(B)]=\text{Mic-Int}[f^{-1}(B)]\) for all \(B \subseteq V\).
Proof: (i) Necessity: Let \( f \) be Strongly Micro SP-continuous and \( F \in \text{Mic SP-Cl}(V,Y) \). That is, \( (V - F) \subseteq \text{Mic SP-O}(V,Y) \). Since \( f \) is Strongly Micro SP-continuous, \( f^{-1}(V - F) \subseteq \text{Mic-O}(U,X) \), and \( (U - f^{-1}(F)) \subseteq \text{Mic-O}(U,X) \). Hence \( f^{-1}(F) \subseteq \text{Mic-CL}(U,X) \).

Sufficiency: Let \( f^{-1}(F) \subseteq \text{Mic-CL}(U,X) \), for all \( F \in \text{Mic SP-CL}(V,Y) \). Let \( B \subseteq \text{Mic SP-O}(V,Y) \). Then \( (V - B) \subseteq \text{Mic SP-CL}(V,Y) \). Then \( f^{-1}(V - B) \subseteq \text{Mic-CL}(U,X) \), that is \( (U - f^{-1}(F)) \subseteq \text{Mic-CL}(U,X) \) implies \( f^{-1}(F) \subseteq \text{Mic-O}(U,X) \). Hence the inverse image of every Micro SP-open set in \( V \) is Micro-open in \( U \). Therefore \( f \) is strongly Micro SP-continuous on \( U \).

(ii) Necessity: Let \( f \) be strongly Micro SP-continuous and \( B \subseteq \text{Mic SP-open} \) set in \( V \). If \( f^{-1}(B) = \emptyset \), then \( f^{-1}(B) \) is Micro-open in \( U \). If \( f^{-1}(B) \neq \emptyset \), then there exists a Micro-open set \( A \subseteq U \) containing \( x \) such that \( f(A) \subseteq B \) which implies \( x \in A \subseteq f^{-1}(B) \) and hence \( f^{-1}(B) \) is Micro SP-open.

Sufficiency: Let \( B \subseteq \text{Mic SP-open} \) set in \( V \) containing \( f^{-1}(x) \), then \( x \in f^{-1}(B) \) and by hypothesis \( f^{-1}(B) \) is a Micro-open set in \( U \) containing \( x \), so \( f(f^{-1}(B)) \subseteq B \). Hence \( f \) is Strongly Micro SP-continuous.

(iii) Necessity: If \( f \) is Strongly Micro SP-continuous and \( F \subseteq V \), \( \text{Mic SP-Cl}(F) \subseteq \text{Mic SP-CL}(V,Y) \) and from (i) \( f^{-1}(\text{Mic SP-Cl}(F))) = f^{-1}(\text{Mic SP-Cl}(F))) \subseteq \text{Mic-CL}(U,X) \) and \( \text{Mic-Cl}(f^{-1}(\text{Mic SP-Cl}(F))) = f^{-1}(\text{Mic SP-Cl}(F))) \). Since \( F \subseteq \text{Mic SP-Cl}(F) \) which implies \( f^{-1}(F) \subseteq f^{-1}(\text{Mic SP-Cl}(F))) \) implies that \( \text{Mic-Cl}(f^{-1}(F)) \subseteq \text{Mic-Cl}(f^{-1}(\text{Mic SP-Cl}(F))) = f^{-1}(\text{Mic SP-Cl}(F))) \). Hence \( \text{Mic-Cl}(f^{-1}(F)) \subseteq (f^{-1}(\text{Mic SP-Cl}(F))) \).

Sufficiency: Let \( \text{Mic-Cl}(f^{-1}(F)) \subseteq f^{-1}(\text{Mic SP-Cl}(F)) \), for every \( F \subseteq V \). Let \( F \subseteq \text{Mic SP-CL}(V,Y) \). Then \( \text{Mic SP-Cl}(F) = F \). By assumption, \( \text{Mic-Cl}(f^{-1}(F)) \subseteq f^{-1}(\text{Mic SP-Cl}(F)) = f^{-1}(F) \). Thus \( \text{Mic-Cl}(f^{-1}(F)) \subseteq f^{-1}(F) \). Hence every Micro SP-closed set \( F \subseteq V \) is Micro SP-closed on \( U \). Therefore \( f \) is Strongly Micro SP-continuous on \( U \).

(iv) Necessity: Let \( f \) be strongly Micro SP-continuous and \( B \subseteq V \). Then \( \text{Mic SP-Int}(B) \subseteq \text{Mic SP-O}(V,Y) \). Thus \( f^{-1}(\text{Mic SP-Int}(B))) \subseteq \text{Mic-O}(U,X) \). That is \( f^{-1}(\text{Mic SP-Int}(B)) = \)
4. Micro SP-homeomorphism

Definition 4.1. Let \((U, \tau_R(U), \mu_R(U)), (V, \tau_R(V), \mu_R(V))\) be two Micro topological spaces. A bijective function \(f: U \to V\) is said to be Micro SP-homeomorphism if \(f\) and \(f^{-1}\) are both Micro SP-continuous.

Example 4.2. Let \(U = \{a,b,c,d\}\) with \(U|R = \{\{a\},\{b,c\},\{d\}\}\) and \(X = \{b,d\}\). Then \(\tau_R(X) = \{U, \phi, \{a\},\{b,c\},\{d\}\}\). If \(\mu = \{a\}\) then \(\mu_R(X) = \{U, \phi, \{a\}, \{b,c\},\{d\}\}\).

Also let \(V = \{1,2,3,4\}\) with \(V|R = \{\{1\},\{2,4\}\}\) and \(Y = \{2,4\}\). Then \(\tau_R(Y) = \{V, \phi, \{2,4\}\}\). If \(\mu = \{2\}\) then \(U,R(Y) = \{V, \phi, \{2\}, \{2,4\}\}\). If \(\mu = \{1,2,4\}\) then \(U,R(Y) = \{V, \phi, \{2\}, \{1,2,4\}\}\). If \(\mu = \{3,4\}\) then \(U,R(Y) = \{V, \phi, \{2\}, \{3,4\}\}\).
Define a bijective function \( f: U \to V \) as \( f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 3 \). Here \( f^{-1}([2]) = \{a\}, f^{-1}([3]) = \{d\}, f^{-1}([1,4]) = \{b,c\}, f^{-1}([2,3]) = \{a,d\}, f^{-1}([1,2,4]) = \{a,b,c\}, f^{-1}([1,3,4]) = \{2,3,d\} \) and \( f^{-1}(V) = U \). Therefore \( f \) is Micro \( SP \)-continuous. Also, here \( f(\{a\}) = \{2\}, f(\{d\}) = \{3\}, f(\{b,c\}) = \{1,4\}, f(\{a,d\}) = \{2,3\}, f(\{a,b,c\}) = \{1,2,4\}, f(\{b,c,d\}) = \{1,3,4\} \) and \( f(U) = V \). Thus \( f^{-1} \) is Micro \( SP \)-continuous. Here both \( f \) and \( f^{-1} \) are Micro \( SP \)-continuous. Hence \( f \) is Micro \( SP \)-homeomorphism.

**Definition 4.3** Let \( (U, \tau(X), \mu(X)), (V, \tau(Y), \mu(Y)) \) be two Micro topological spaces. Then a bijective function \( f: U \to V \) is said to be Strongly Micro \( SP \)-homeomorphism if \( f \) and \( f^{-1} \) are both Strongly Micro \( SP \)-continuous.

**Example 4.4** Let \( U = \{a,b,c,d\} \), with \( U|\tau = \{\{a,d\}, \{b\}, \{c\}\} \) and \( X = \{c,d\} \). Then \( \tau(X) = \{U, \phi, \{c\}, \{a,d\}, \{a,c,d\}\} \). If \( \mu = \{b\} \) then \( \mu(X) = \{U, \phi, \{b\}, \{c\}, \{a,d\}, \{b,c\}, \{a,b,d\}, \{a,c,d\}\} \). Micro \( SP \)-open sets in \( U \) are \( \{U, \phi, \{b\}, \{c\}, \{a,c,d\}\} \). Also let \( V = \{1,2,3,4\} \) with \( V|\tau = \{\{1\}, \{2,3\}, \{4\}\} \) and \( Y = \{2,4\} \). Then \( \tau(Y) = \{V, \phi, \{4\}, \{2,3\}, \{2,3,4\}\} \). If \( \mu = \{1\} \) then \( \mu(Y) = \{V, \phi, \{1\}, \{4\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{2,3,4\}\} \) which are Micro-open sets in \( V \).

Define a bijective function \( f: U \to V \) as \( f(a) = 2, f(b) = 1, f(c) = 4, f(d) = 3 \). Here \( f^{-1}(\{1\}) = \{b\}, f^{-1}(\{4\}) = \{c\}, f^{-1}(\{1,4\}) = \{b,c\}, f^{-1}(\{2,3\}) = \{a,d\}, f^{-1}(\{1,2,3\}) = \{a,b,d\}, f^{-1}(\{2,3,4\}) = \{a,c,d\} \) and \( f^{-1}(U) = V \). Thus \( f \) is Strongly Micro \( SP \)-continuous. Also, here \( f(\{b\}) = \{1\}, f(\{c\}) = \{4\}, f(\{b,c\}) = \{1,4\}, f(\{a,b,d\}) = \{1,2,3\} \) and \( f(U) = V \). Therefore \( f^{-1} \) is Strongly Micro \( SP \)-continuous. Here both \( f \) and \( f^{-1} \) are Strongly Micro \( SP \)-continuous. Hence \( f \) is Strongly Micro \( SP \)-homeomorphism.

5. **Conclusion**

In this paper, we studied some more properties of Micro \( SP \)-continuous functions and expounded their relations with Strongly Micro \( SP \)-continuous and Strongly Micro \( SP \)-open map. Also we launched a Micro \( SP \)-homeomorphism concepts. Furthermore, this work will be extended to establish the new concepts of Micro \( SP \)-irresolute, Micro \( SP \)-open map and Micro \( SP \)-closed map and their relative properties will be derived. In future, Micro \( SP \)-homeomorphism concepts can be applied in real life situations through some Micro topological structures.
6. References


